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THE DESIGN OF A MOLTEN FUELED, VORTEX REACTOR, FOR USE AS A ROCKET PROPULSION DEVICE

by

Robert Philip Schmitt

A

THESIS

submitted to the faculty of the

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Approved by

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ABSTRACT

The purpose of this investigation is to design a nuclear reactor which can be used as a rocket propulsion device. The reactor investigated is fueled with a molten uranium-bismuth alloy and utilizes a two phase vortex flow principle. Hydrogen is employed as a coolant and rocket propellant. The hydrogen is heated directly by the fuel thus eliminating the need for heat exchangers.

The major part of the investigation consists of a two group neutron diffusion study to show the operational feasibility of this type of reactor and to obtain an optimum design.

The study shows that this type of reactor is practical and that it can be used successfully as a rocket propulsion device.

ACKNOWLEDGEMENTS

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I. INTRODUCTION

This thesis proposes nuclear power as an alternative to chemical rocket propulsion.

The exploration of outer space appears, at the present time, to be limited by the performance capabilities of the present chemical rockets. The excessive mass of the present chemical rockets and the limited specific impulse restrict them to orbital flights or short missions with light payloads. One answer to this problem is the development of a nuclear rocket engine. The propulsion system developed in this thesis has a much smaller mass than a chemical rocket and has more than twice the specific impulse. Two areas were considered in the reactor design. A two group neutron diffusion study was done first to insure that the reactor was feasible. A commercial computer code, Exterminator II, was used to perform the two dimensional neutron calculations. Various parameters were changed to obtain the optimum design. These included; reactor dimensions, types of materials used, and amounts of these various materials. Once the reactor was shown to be practical from a nuclear viewpoint, a further study of mechanical and chemical properties was made. Investigation in these areas showed that the reactor could be used as a rocket engine.

II. LITERATURE REVIEW

Chemical rockets, which have proved quite capable of performing simple orbital or short range missions, are definitely limited in their future possibilities. This is because the energy which can be released by chemical combustion is restricted by weakness of the chemical bond (1).

In order to successfully compare two types of rockets, a common parameter must be found. One commonly accepted method is to compare burnout velocities, a high burnout velocity being desirable. The burnout velocity is measured with respect to the ratio of initial rocket mass to the burnout mass. Richard Plebuck (2) gives the following expression for burnout velocity:

$$V_b = I_{sp}\, g \quad (m_o/m_b) - g\, t_b + V_o \;, \qquad 2.1$$

where:

t = Time	g = 32.17 ft/sec^2
V = Velocity	o = Initial state
m = Rocket mass	b = Burnout state

I_{sp} = Specific impulse

Assuming the burning time and initial velocities of the two systems are equal, then the burnout velocity is only a function of mass and specific impulse. Specific impulse, defined as the engine thrust per unit mass flow rate of propellant will then be used to roughly compare rocket systems since systems masses vary widely.

The specific impulse of a chemical rocket must be less than 400 seconds (1). In order to overcome the low impulse rates, chemical rockets have been multistaged and designed with extreme precision. If a system of higher impulse were designed, not only would the overall performance increase but both the excessive size and exact design limits could be reduced. The nuclear powered rocket is one answer to this problem.

A number of designs for nuclear rockets have been theorized or proposed. However, the development of such systems has rarely gone beyond the basic design stage.

Nuclear propulsion systems may be classified into two areas. These are by the method of converting fission energy into a propulsive force and by the phase or state of the fuel employed.

Three methods of nuclear energy conversion have been considered. Two of these would use the energy produced in the reactor in some kind of secondary system. A nuclear-electric system would employ the fission energy to create either an electric or magnetic field which would heat the propellant. A second possible method would be to employ a heat exchanger or similar device to transfer energy to the propellant. Both of these systems would require a large amount of extra equipment which would increase the system's mass but would not add directly to the power produced. The third method, a direct conversion system, would eliminate any superflous equipment. The propellant would

also serve as the reactor coolant and would be heated on a single or multiple pass through the reactor core. Assuming that the amount of heat transferred to the propellant would be at least equal to the amount transferred in a heat exchanger system, the direct transfer system would have a definite advantage. The phase of the fuel employed is one of the more important factors in determining the capability of a nuclear rocket. The specific impulse of a rocket operating in a vacuum may be expressed as

$$I_{SP} = \nu_v \left[\left(\frac{2\gamma}{\gamma-1}\right) \frac{R_u}{Mg_o} \; T_c \; \eta_c + \left(\frac{V_c}{g_o}\right)^2 \right]^{\frac{1}{2}} \qquad 2.2$$

$$+ \left[\frac{1}{\nu_d} \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}} \left(\frac{\gamma-1}{2\gamma} \; \frac{R_u}{Mg_o} \; \frac{T_c}{\eta_c} \right)^{\frac{1}{2}} \right]$$

Where:

ν_v = Velocity coefficient V_{ea}/V_{ei}

γ = Ratio of specific heats

R_u = Universal gas constant

M = Molecular weight of propellant

V_{ea} = Actual exit velocity

T_c = Propellant temperature

V_c = Propellant velocity

ν_d = Ratio of actual to theoretical flow

$\frac{P_e}{P_c}$ = Nozzle pressure ratio, exit/chamber

η_c = Nozzle efficiency

V_{ei} = Ideal exit velocity

P_e = Exit pressure

P_c = Chamber pressure

From this expression we can see that there are three dominant factors. A propellant of low atomic weight and high propellant temperatures and velocities are desirable. Since the ratio of nozzle exit to entrance pressure must be less than one and since $\frac{\gamma-1}{2\gamma}$ is also less than one, the first term in equation 2.2 is dominant. Thus the possible attainable temperatures are extremely important in the specific impulse of a rocket.

Although some mention has been made of liquid and gaseous fueled reactors in literature, almost all of the designs proposed have been for solid fueled reactors(2,3). Six solid fuel reactors (KIWI Reactors) have been tested under the government sponsored program "Project Rover" (2). These tests were designed to demonstrate the operational feasibility of solid core rocket reactors. Some of the areas studied were responsed to reactivity changes, coolant flow rate, coolant temperature, and start up methods. The

first three tests employed gaseous hydrogen while the fourth, KIWI-BIA used liquid hydrogen (4). Although the tests were fairly promising, the project appears to be inactive at this time.

The proposed designs for solid core reactors usually consist of a number of disc or flat plate fuel elements (2, 3,5). The propellant-coolant passes over and between the fuel plates and exhausts through a nozzle. This type of reactor has the advantage of being relatively easy to construct as it is quite similar to some power reactors in operation. The major disadvantage of this system is that operating temperatures are limited by the melting point of the fuel. The maximum fuel temperature of such a reactor appears to be about 2300°C if a uranium-graphite fuel is employed. At these temperatures, however, the hydrogen coolant could never come in direct contact with the fuel as carbon is attached by hydrogen at temperatures above 1500°C (6).

In order to obtain higher operating temperatures and specific impulse, the maximum fuel temperature must be increased. In order to accomplish this, it is necessary to use liquid or gaseous fueled reactors. Of these two possibilities, the gaseous fueled reactor will obviously yield higher impulses. A reactor of this type would only be limited by the temperatures which the structural materials could withstand. The major problem with the gaseous fueled reactor is the loss of fissionable material.

It is found that the coolant will carry away a sizable percentage of the fuel as it leaves the core. The tolerable fuel loss would be one atom per thousand (7). Two possible methods of separation have been proposed (1), a vortex system similar to the one discussed in this report and a magnetic containment device. Neither of these has proved satisfactory, and so while gaseous reactors offer higher temperatures and impulses, they cannot be seriously considered until the fuel-coolant separation problem has been solved.

The liquid fuel reactor serves as an obtainable compromise between the gaseous and solid fueled reactors. The liquid fueled reactor offers operating temperatures as high as 4300°C and presents no unsolvable problem in fuel loss (1). The high operating temperatures have two desirable features. First, specific impulses as high as 1600 seconds are attainable. This would be twice that of a solid fueled reactor and four times that of the most efficient single staged chemical rocket. Secondly, at temperatures above 2000°C and pressures below 30 atm., hydrogen will begin to dissociate. The energy required to break H_2 molecules into H^+ atoms is 10^5 BTU/lb. or 4.4 ev/molecule. Thus, if conditions are favorable, hydrogen dissociation will increase the amount of heat transferred to the coolant. Assuming the hydrogen will recombine in the collection chamber due to three body collisions, an additional source of heat is available. The chemical equations for this

process are shown below

$$1/2\ H_2 + H_d \rightarrow H^+$$
$$2H + x \rightarrow H_2 + x^* + H_d \qquad 2.3$$

Where:

H_d is the heat of disassociation

x is a hydrogen atom or H_2

The separation of the fuel from the coolant in a liquid fueled reactor is accomplished by using a two phase vortex system. The centrifuge action of the vortex causes the heavy fuel to be forced to the outside of the fuel container while the lighter gas is passed inward. Although some experimental work has been done on two phase vortex flow, no references could be found on the specific system used in the reactor. J. F. Lafferty's work provided information on possible velocities, pressures and void fractions (8). However, his work was performed using water and air. Due to the differences in densities of water and uranium, and hydrogen and air, his results could only be used as approximate guidelines for values used in this report.

There has been a good deal of work done in liquid metal fueled reactors (9). All of the systems studied seem to be a heat exchanger type system in which the liquid fuel is passed through a core region and then to a heat exchanger device. This type of system has been investigated in the late fifties and early sixties by the Babcock and Wilcox Company in Akron, Ohio. The Babcock and Wilcox Liquid Metal Fueled Reactor (LMFR) utilized a uranium-bismuth mixture but

that was the only similarity between the reactor proposed in this report and the LMFR.

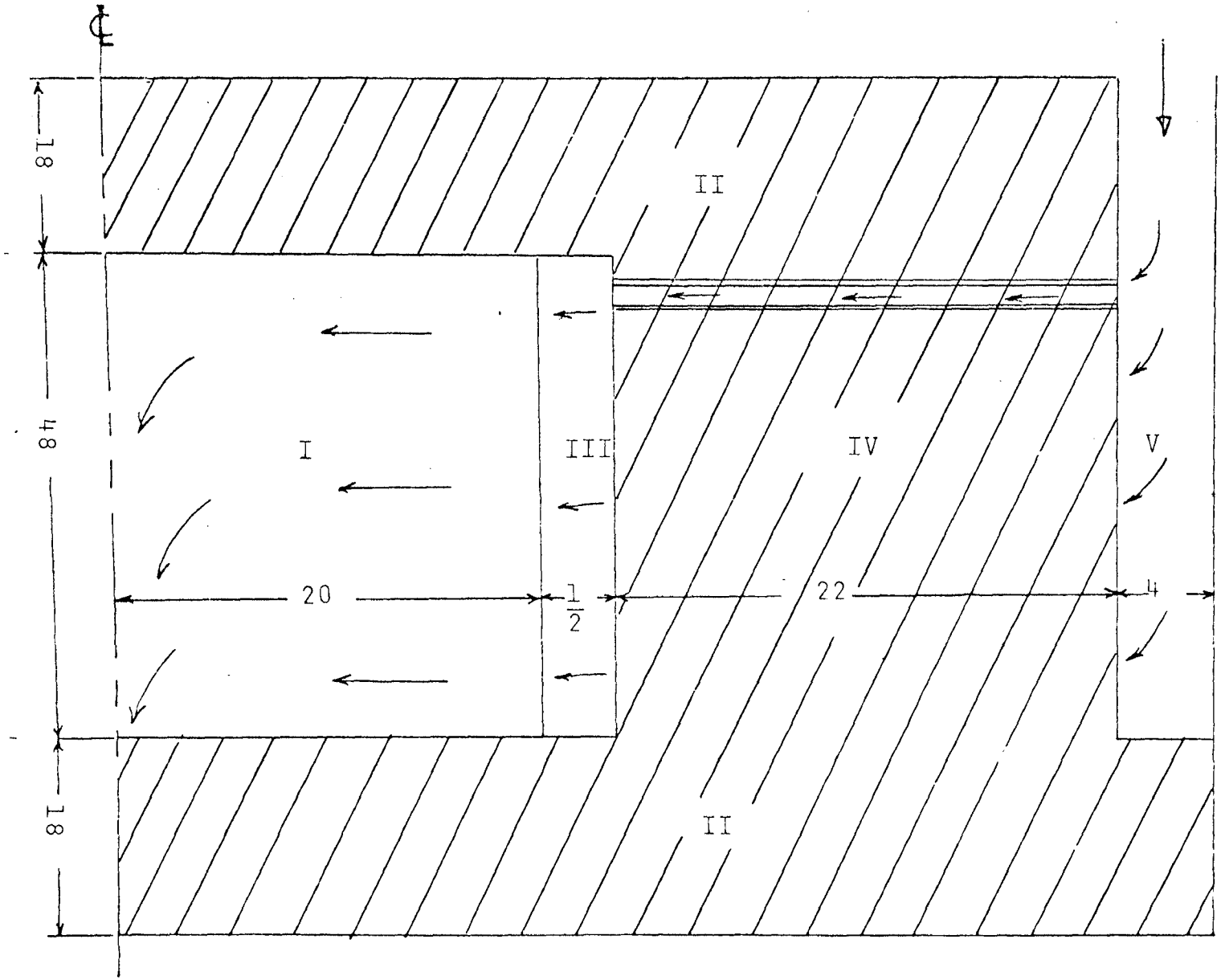
No information could be found on the development of a liquid fueled reactor using the vortex principle.

III. BASIC REACTOR DESIGN

The reactor studied in this report is a molten fueled thermal reactor with a beryllium oxide reflector. A cross sectional view is shown in Figure 1. It should be noted that this is only an initial design. The reactor has a right circular, cylindrical geometry, and is symmetric about the axis. The fuel is a molten alloy of uranium-bismuth and is contained in an annular region one half inch in width. The coolant employed is gaseous hydrogen and is injected into a plenum near the outside of the reactor, (shown by arrows in Figure 1). The hydrogen will be stored in the liquid state and evaporized before entering the reactor. The coolant leaves the plenum through tubes in the reflector. Each of these tubes leads to a nozzle which ends at the fuel reflector interface. The nozzles increase the coolant's velocity and change its direction so it is injected tangential to the liquid fuel. This causes the molten fuel and hydrogen to form a two phase vortex. The centripetal force created by spinning the fuel forces the hydrogen to diffuse through the liquid where it is collected in a centrally located chamber. From here it is passed through a DeLaval nozzle producing the rocket thrust (not shown in Figure 1).

The reactor was divided into five regions which are: I. collection chamber, II. reflector region, III. fuel regions, IV. reflector plus hydrogen region, and V. hydrogen inlet region.

Fig. 1 Cross Sectional View of the Reactor - Initial Design



All dimensions given in inches.

As the hydrogen passes through region IV, it absorbs heat from the reflector and its density decreases as a result. This in turn will cause a decrease in the macroscopic cross sections which are proportional to the density. In order to account for this change, the highest values of the cross sections were used. A larger number of subdivisions would be desirable; however, the limitations of available core storage prohibited it.

IV. REACTOR PHYSICS

The major emphasis of this thesis is on reactor physics. As no other reactor has been designed which utilizes the vortex principle, hydrogen cooling, and a liquid fueled core, the basic nuclear physics of the system must be studied. The ultimate aim of this study is to prove that the proposed reactor can achieve criticality and to determine the neutron fluxes.

A. Diffusion Theory

Two methods are available for studying the reactor diffusion or transport theory. Of the two, diffusion theory was used.

Neutron diffusion is considered to vary only with the radius and height of the reactor. All calculations are to be done under steady state conditions.

Exterminator II, a computer code which solves two dimensional multigroup diffusion equations, was used in this study (10). This code, developed at Oak Ridge National Laboratory, provides an approximate solution to otherwise unsolvable diffusion equations.

The necessity for such a computer code can easily be seen by briefly looking at a set of two group, two region diffusion equations.

fuel -1

fast -1

$$D_{11}\nabla^2\phi_{11} - \sum_{r_1}\phi_{11} = \nu\sum_{1f}\phi_{11} + \nu\sum_{2f}\phi_{12}$$

thermal -2

$$D_{12}\nabla^2\phi_{12} - \sum_{r_2}\phi_{12} + \sum_{1s}\phi_{11} = 0$$

reflector region -2

fast -1

$$D_{21}\nabla^2\phi_{21} - \sum_{r_1}\phi_{21} = 0$$

thermal -2

$$D_{22}\nabla^2\phi_{22} - \sum_{r_2}\phi_{22} + \sum_{s_1}\phi_{11} = 0$$

For convenience, the equations are transformed into cylindrical coordinates as shown below.

$$\frac{1}{r}\frac{d}{dr}\left(rD_{11}\frac{d\phi_{11}}{dr}\right) + \frac{d}{dz}\left(D_{11}\frac{d\phi_{11}}{dz^2}\right) - \sum_r\phi_{11} = \nu\sum_{1f}\phi_{11} + \nu\sum_{2f}\phi_{12}$$

It should be noted that the diffusion coefficient is a function of both radius and height.

Assuming

$$\Phi(r,Z) = \phi_r(r) \cdot Z_c(Z)$$

an attempt is made to separate the valuables, resulting in the following for region I

$$\frac{1}{\phi_{11}D_{11}}\left[\frac{1}{r}\,d\left(\frac{\phi_{11}D_{11}}{dr}\right) + \frac{d}{dr}\left(D\,\frac{d\phi_{11}}{dr}\right)\right] + \frac{1}{Z_c}\,\frac{d^2Z_c}{dZ^2}$$

$$-\frac{\sum_{1r}}{D_{11}} = \frac{\nu\sum_{1f}}{D_{11}} + \frac{\nu\sum_{2f}\phi_{12}Z_{12}}{D_{11}\phi_{11}Z_{11}}$$

$$\frac{1}{\phi_{12}D_{12}}\left[\frac{1}{r}\,d\left(\frac{\phi_{12}D_{12}}{dr}\right)+\frac{d}{dr}\left(D_{12}\,\frac{d\phi_{12}}{dr}\right)\right]$$

$$+\frac{1}{Z_c}\,\frac{d^2Z_c}{dZ^2}-\frac{\Sigma_{2a}}{D_{12}}+\frac{\Sigma_{13}\phi_{11}Z_{11}}{D_{12}\phi_{12}Z_{12}}=0$$

It is obvious that no exact solution can be attained for the diffusion equations in cylindrical geometry and some approximate technique must be used. The availability of Exterminator II and its wide variety of options were the major reasons for its selection.

B. Flux Distribution

The flux distribution in a bare finite cylindrical reactor is $AJ_o\ (2.405r/R_c)\ \cos\ (\pi z/H_c)$

Where: A = Constant

R_c = Critical Radius

H_c = Critical Height

Thus, one should expect a radial flux distribution which approximates a zero order Bessel function and an axial flux distribution resembling a cosine function. The presence of a reflector will, of course, alter these shapes. A thermal spike or peak will appear at or near the fuel reflector interface due to the return of thermal neutrons to the core. This peak is a function of both the reflector thickness and its slowing down properties.

The flux distribution in the reactor under study can

be expected to vary considerably from the bare cylinder or simple reflected cylinder. The main reason for this difference is that the reactor core is poorly moderated. Although the core contains 50% hydrogen, the coolant is an ineffective moderator. This is due to the low hydrogen density (.002 lb/ft^3). Thus, only a few neutrons will be thermalized by the hydrogen. The uranium and bismuth are also poor moderators because of their large atomic weight. It is reasonable to assume that most of the neutrons will be thermalized in the reflector. A portion of these will find their way back to the fuel and will cause fission. Because of this, the highest thermal flux should be in the reflector, near the fuel-reflector interface. The major advantage of this system is that most of the neutrons are thermalized outside of the fuel area and they will escape resonance absorption by the uranium and bismuth. The thermal neutron flux should have its lowest value somewhere near the center of the reactor, increase as it approaches the fuel region and then increase as the radius of the fuel region increases, reach its highest point in the reflector and then decrease.

The fast neutron flux will not be directly affected by the presence of the reflector. The fast flux should be highest in the fuel, near the reflector. This is because the thermal flux is highest here and consequently the fission rate is also. The fast flux should be lowest in the reflector.

V. REACTOR MATERIALS

The initial design of the reactor calls for the use of three basic materials; hydrogen, beryllium oxide, and a molten uranium alloy. A number of other materials will undoubtedly be present in the actual working reactor. However, for the initial design developed here they will not be considered.

A. Coolant-Propellant

Hydrogen was selected as the coolant-propellant over a number of other choices which included; water, various alcohols, hydrocarbons, air and nitrogen compounds. If for no other reason, hydrogen was an obvious choice because of its low molecular weight. According to Bussard and DeLauer, "The energy content per unit mass of working fluid is evidently inversely proportional to the atomic mass of the working fluid used" (1). This can be seen from equation where the specific impulse decreases as the molecular weight increases, all other factors being constant. Since the lowest possible molecular weight in chemical propellants is nine (ozone-hydrogen system) (12) a rocket employing hydrogen has an extreme advantage.

High molecular weight was the main reason for ruling out the other propellants mentioned, however, there were other reasons for selecting hydrogen. As mentioned before, hydrogen molecules are unstable at temperatures above 4000°F and will dissociate into H^+ atoms and its heat

of dissociation is much greater than other propellants. Secondly, hydrogen has a higher heat capacity (4.2 BTU/lb°R) than any other gases mentioned (1). The heat capacity is more than twice that of helium and CH_4 (1.8 and 1.6 BTU/lb°R). Hydrogen also enjoys the highest thermal conductivity (4.0 BTU/hr-ft-°F). One disadvantage of hydrogen is that at high temperatures it is a strong reducing agent, reacting with carbon and some metals. Because of this, the use of a graphite reflector or fuel mixture was ruled out. Since the coolant passes through the fuel mixture, it should also have reasonable nuclear properties. Hydrogen has essentially a zero fast neutron absorption cross section and a reasonably small one for thermal neutrons (.29 barns). It should also be noted that the average increase in lethargy per collision (ξ) is unity for hydrogen. This means that a neutron can lose all of its energy in a single collision with hydrogen. Even though the amount of hydrogen in the fuel may be small, the fractional energy loss will be of significance in a poorly moderated core.

B. Reflector

The choice of hydrogen as the propellant restricted the choice of a reflector material. A graphite reflector was immediately ruled out because of the hydrogen presence. The three main criteria set for the reflector were light weight, and temperature resistance, and of course, desirable nuclear properties.

With the elimination of carbon, beryllium or beryllium

oxide remain as the two choices with low molecular weights. Pure beryllium was considered first. Of course, its most desirable property is its low density (1.85gm/cm^3). It is reasonably strong at high temperatures, its short-time tensile strength is 6000 lb/in^2 at 2000°R and has a Young's modulus of 34-37 x 10^6 lb/in^2 at the same temperature. The thermal conductivity is high, being fifty four BTU/hr-ft-°F at 1500°R. Another desirable feature is its low coefficient of linear expansion (11 x 10^{-6}/°F) at 2000°R. The excellent moderating properties are quite familiar and need not be mentioned. Pure beryllium has one disadvantage, however, its low melting point (2800°R). If the reflector is to be in contact with the fuel at a proposed operating temperature between 3000 & 4000 °R, beryllium cannot be used. It is a possibility, however, if a material with a high melting point, such as tungsten, is placed between the reflector and fuel.

Upon investigating the various properties of beryllium oxide, it was found that this material is the best of the choices available. Some of the properties of beryllium oxide are listed in Table I. These appear to be quite reasonable except for the tensile and compressive strength. These values are low and would necessitate the use of some reinforcing device. This could be in the form of steel bands or a steel container around the outer circumference of the reflector. While beryllium oxide has a fairly high melting point, the maximum operating temperature of the reactor may be limited by it. It should be noted that most

Table I

Properties of Beryllium Oxide

Density = 2.9 gm/cm^3

Melting Point = 5050 °R

Thermal conductivity = 8.5 BTU/hr-ft-°F at 3000°R

Thermal conductivity = 21 BTU/hr-ft-°F at 1500 °R.

Coefficient of linear expansion = 16.0×10^{-6} $°F^{-1}$ at 2400°R

Tensile strength = near zero above 2500° R

Compressive strength = 7950 lb/in^2 at 2700°R

Table I

Properties of Beryllium Oxide

Density = 2.9 gm/cm^3

Melting Point = 5050 °R

Thermal conductivity = 8.5 BTU/hr-ft-°F at 3 000°R

Thermal conductivity = 21 BTU/hr-ft-°F at 1 500 °R

Coefficient of linear expansion = 16.0×10^{-6} $°F^{-1}$ at 2400°R

Tensile strength = near zero above 2500° R

Compressive strength = 7950 lb/in^2 at 2700°R

of the work with beryllium oxide has been done at temperatures below 3600°F which may be below the temperature in the reactor.

Further investigation was necessary in order to show that beryllium oxide was compatable with the other reactor materials. It has been reported that beryllium oxide does not interact with hydrogen (13). Once again the temperature range investigated was up to 3600°F. Beryllium oxide does react with uranium, however. It has been reported that the system will form an oxide of uranium, beryllium-uranium solution and an oxide deficient substrate (14). However, since the reactor operating time for any given rocket maneuver will be short, this should not be a major factor.

A secondary system would employ a tungsten fuel container surrounded by the reflector. The tungsten container will protect the reflector from corrosion and may permit higher operating temperatures.

The use of a tungsten container would also eliminate another disadvantage of a beryllium oxide fuel interface. Since beryllium oxide is a porous material, there is a possibility of the fuel diffusing through the reflector. The tungsten container would prevent this. Natural tungsten could not be used for the container as it has a thermal microscopic absorption cross section of 19 barns. Other materials with high melting points such as osmium, rhenium, and tantalum have large cross sections also. Only niobium

molybdenum, and tungsten-184 have cross sections small enough to be considered. Of these, W-184 was considered best with molybdenum second. Since the reactor proposed in this study operates below the melting point of beryllium oxide, no calculations were performed with a tungsten container.

C. Fuel

The major criterion in choosing a fuel mixture for the reactor was temperature resistance. It was desirable to select a fuel with a high boiling point, but a fairly low melting point which will provide a wide operating range. It was decided that highly enriched uranium in a molten metal carrier would be the most likely choice. A number of possibilities were considered. These were uranium with titanium, tungsten, molybdenum, niobium, carbon, bismuth and beryllium. Phase diagrams of these alloys are found in the Reactor Materials Handbook (15). Of these, three were eliminated immediately, titanium and tungsten because of high thermal neutron absorption and carbon because of its reaction with hydrogen. Molybdenum and niobium offered the possibility of high working temperatures. However, the melting points of these alloys were high. An initial estimate of the fuel composition was a maximum of twenty per cent uranium and a minimum of eighty per cent carrier. For these compositions the liquid phase of the niobium and molybdenum alloys exist only above 4200 and 4500°F respectively.

If it is found that only a small amount of fissionable

material (3%) is necessary to reach critically a uranium-beryllium fuel system could be considered. There is a reasonable interval between the melting and boiling temperatures at 97% beryllium and above 99% and beryllium has good neutron moderating properties. One problem which might render this alloy useless is the large difference in density of the constituents. The centrifugal action of the vortex might separate the fuel from the carrier and the beryllium could be carried off by the hydrogen.

The uranium-bismuth system offers the widest operating range of the alloys considered. For a bismuth content above 50%, the alloy melts at or below 1800°F and does not boil below 3500°F. No definite information could be obtained regarding the actual boiling point of the alloy. It will be assumed that the boiling point is above 4000°F. The fuel will be under pressure and this should raise the boiling temperature. While bismuth is not an effective neutron moderator, neither is it a neutron absorber. Bismuth-209 will undergo a (n,γ) reaction to form Bi-210. This has a five day life and decays to Po-210 by beta emission. The polonium in turn undergoes an alpha decay (half life 140 days) to Pb-206. The cross section for this reaction is .016 barns. Since the reactor will operate for short periods of time, the loss of bismuth from the systems will be small.

Two isotopes of uranium were considered, U-235 and U-233. The 235 isotope would be preferable as it is more

available than 233 and therefore, less expensive. At normal temperatures, U-235 has a much higher fission cross section. However, when operating at high neutron temperatures, the fission and absorption cross sections of U-235 decreases while those of U-233 show an increase. Thus, at high temperatures, U-233 may have a higher fission cross section. This will be discussed further in Section VI.

VI. CROSS SECTIONS

The nuclear microscopic cross sections used in the various calculations are based on the LASL-16 group found in ANL-5800 (16). This set was chosen because of the desirable energies of the various neutron groups. All of the elements on the reactor except bismuth were found in the above reference. Cross sections for Bi-209 were approximated from cross section curves in BNL-325 (17) and may contain some degree of error.

The cross sections for beryllium oxide were obtained by adding the macroscopic cross sections of oxygen and beryllium. We know that

$$\Sigma_{total} = \Sigma_1 + \Sigma_2 + \cdots + \Sigma_n \qquad 6.1$$

and that

$$\Sigma = N_o \sigma \qquad \text{with } N_o = \rho N_a / M$$

Where: ρ = Density gm/cm^3

M = Molecular weight

$N_a = 6.03 \times 10^{23}$ atoms/cm^3

Thus

$$N_o \sigma_{BeO} = N_o \sigma_{Be} + N_o \sigma_O \qquad 6.2$$

Since the number of oxygen and beryllium atoms in the compound are the same, equation 6.2 simply reduces to

$$\sigma_{BeO} = \sigma_{Be} + \sigma_O \qquad 6.3$$

As capture or absorption cross sections were not listed for U-235, U-238 and Bi, they were calculated from the following relationship

$$\sigma_{tr} = \sigma_a + \sigma_s (1 - \bar{\mu}) \qquad 6.4$$

Where:

$\bar{\mu}$ = the average cosine of the scattering angle (11).

Since $\bar{\mu}$ decreases with increasing mass, (.00725 for uranium) the absorption cross sections is essentially

$$\sigma_{tr} - \sigma_s = \sigma_a \qquad 6.5$$

It was originally intended to use sixteen neutron groups which would in turn be reduced to three broad groups by using option 14 of the Exterminator II code. However, the limitations of available core on the IBM 360/50 computer made it necessary to reduce the number of energy groups to ten. This was accomplished by averaging the group cross sections with respect to energy. Groups 1 and 2; 6, 7, 8, and 9; and 10, 11, and 12 were averaged into three groups. These pairings were chosen because the changes in lethargy were approximately the same.

As mentioned above, option 14 was to be used to generate three broad groups, one from 100 ev to ∞, one from .1 ev to 100 ev, and one thermal group of energy less than .025 ev. Unfortunately, this method does not easily apply itself to a reactor design problem. Since the broad group cross sections are a function of nuclide concentration and region volume, they would have to be recomputed for each different reactor design. This would necessitate the use of an additional thirty to forty hours of computer time. In order to avoid this, a simple weighted average was used

to reduce the ten neutron groups to three. The three group cross sections obtained by this method are listed in Table II. This method was checked by comparing the averaged three group cross sections for U-235 to those given in ANL-5800. It was found that the averaged values were usually within .1 barns of the given values.

Due to high temperature expected in the reactor, the thermal neutron cross section cannot be considered Maxwellian. The cross sections given in BNL-325 and in ANL-5800 were computed at 293°K and will have to be corrected. Meghreblian and Holmes give the following method for finding the effective neutron temperature (18).

$$T_n = T_N (1 + 1.111\ AK) \qquad 6.6$$

Where:

T_N = Moderator temperature (°K)

T^* = 293°K

N_a = Nuclear density of absorbers

N_s = Nuclear density of scatterers

$$K = \left(\frac{2T^*}{3T_n}\right)^{\frac{1}{2}} \left(\frac{N_A}{N_s}\right)\left(\frac{\sigma_a(T^*)}{\sigma_s}\right) \qquad 6.7$$

In all cases K was found to be small so the neutron temperature will be approximately that of the moderator. Since the absorption cross sections of hydrogen, oxygen, and beryllium were small(<.0095 barns) the reflector was treated as a pure scatterer (i.e. $N_a = 0$).

The effective cross section for 1/v absorbers can be

Table II. Three Group Neutron Cross Sections

U-235

GP.	σ_a	σ_f	σ_{tr}	ν	$\sigma_{i\ i+1}$	$\sigma_{i\ i+2}$
1	1.21	1.23	4.50	2.82	.05	0.0
2	66.5	44.0	76.2	2.38	.04	-
3	611	516	621	2.45	-	-

H_2

GP.	σ_a	σ_f	σ_{tr}	ν	$\sigma_{i\ i+1}$	$\sigma_{i\ i+2}$
1	0.0	-	8.67	-	.10	.096
2	.02	-	6.69	-	2.60	-
3	.29	-	7.09	-	-	-

Bi

GP.	σ_a	σ_f	σ_{tr}	ν	$\sigma_{i\ i+1}$	$\sigma_{i\ i+2}$
1	.05	-	1.44	-	.01	0.0
2	0.0	-	5.46	-	0.0	-
3	.009	-	9.21	-	-	-

BeO

GP.	σ_a	σ_f	σ_{tr}	ν	$\sigma_{i\ i+1}$	$\sigma_{i\ i+2}$
1	.063	-	3.96	-	.98	0.0
2	0.0	-	9.10	-	1.21	-
3	.009	-	9.21	-	-	-

All cross sections given in barns.

found from the following equation (19).

$$\sigma = \left(\frac{\sqrt{\pi}}{2}\right)\left(\frac{T^*}{T}\right)^{\frac{1}{2}} \sigma(E_o) \qquad 6.8$$

A typical calculation for hydrogen at 2000°F gives the effective cross section as .14 barns. This is about half of the value at 293°K (.29 barns).

For the purposes of the calculations performed in this paper, the higher or uncorrected values of the absorption cross sections will be used. The purpose of this is to take into account the presence of structural materials, control devices, impurities, etc., which are not included in the actual calculations. It is not expected that the higher values will duplicate the effects of these materials. However, the overdesign which will result, will allow to some degree for the inclusion of foreign bodies.

Corrections for U-235, a non 1/v absorber, were taken from Westcotts work (20). His values of the effective absorption and fission cross sections at 293°K are higher than those given in ANL-5800. For example, the absorption cross section in ANL-5800 is 611 barns, while Westcotts value is 676 barns. At the neutron temperature encountered in the fuel, Westcotts cross sections are essentially equal to those given in ANL-5800. As a result, the cross sections used were:

σ_a = 611 barns

σ_f = 516 barns

VII. REACTOR DESIGN

The first basic steps in designing the reactor have already been taken. Materials have been chosen for the fuel, reflector, and coolant, and the size of the reactor has been set. Two alternatives are available for the next step. Either the amount of fuel or the effective multiplication factor may be chosen and the other calculated. For a reactor type which has been previously studied, a reasonable guess may be made at the fuel concentration and the corresponding k_{eff} calculated. The fuel concentration can then be adjusted until the desired multiplication is found.

For a new reactor design it might be difficult to choose fuel concentrations which are reasonably close to those necessary for criticality. In this case, the desired k_{eff} can be selected and the corresponding fuel concentrations found. Exterminator II offers the user both of these options.

The first choice was to use the nuclide search, for a given multiplication factor, option. A multiplication factor of 1.05 was selected as were the following nuclide concentrations.

$$N_{25} = 5.60 \times 10^{-3} \text{ atoms/cm}^3$$

$$N_H = 1.26 \times 10^{-5} \text{ atoms/cm}^3$$

$$N_{Bi} = 1.05 \times 10^{-2} \text{ atoms/cm}^3$$

From Lafferty's work on vortex flow, a void fraction of 50% was assumed in the core. The void refers to the absence of liquid fuel and is actually filled with hydrogen. The remaining volume was assumed to contain 75% bismuth, 25% U-235, and 0% U-238.

A good deal of difficulty was encountered in using Exterminator's nuclide search. While the multiplication factor converged (ε = .0001) in less than thirty iterations, the flux failed to converge in three hundred iterations. When the convergence criterion was reduced, the flux converged. However, the code performed another iteration after convergence. This extra iteration changed k_{eff} considerably and the results were worthless.

The failure of the flux to converge can be explained in a number of ways which include:

1) Not enough mesh points were chosen
2) The flux was not well behaved
3) The initial fuel concentration guess was grossly in error

Because of the difficulty with option 14 of Exterminator, the assumed nuclide concentrations were used in a search for k_{eff}. This resulted in a three group multiplication factor of .0517.

It should be noted that there is some doubt to the actual magnitude and shape of the fluxes obtained using Exterminator II. This uncertainty is due to the failure of

the neutron flux to converge. In order to check the accuracy of the results, the same problem was solved using AIM-5, a one dimensional computer code. The results shows that while the flux distribution had the same shape in both cases, the fluxes obtained in AIM-5 were larger by a factor of 10^2 to 10^3. An obvious explanation for this is the fact that AIM-5 does not consider neutron leakage from the top and bottom of the reactor. This will account for the higher flux values calculated by AIM-5, as Exterminator II considered both top and bottom leakage.

A multiplication factor of .0517 indicated that there was a serious error in the initial reactor specifications. The first area which was investigated was the amount of U-235 in the fuel. Since 233 kilograms yielded a k_{eff} of .0528, trials were made with 116 and 467 kilograms of U-235. This information is depicted graphically in Figure 2. While increasing the amount of U-235 causes a rise in the multiplication factor, it is estimated that two thousand kilograms would be needed to reach criticality.

The neutron fluxes in this case are as predicted. These are shown in Figures 3 and 4. Figure 3, the radial thermal flux, shows the ratio of ϕ max/ ϕ min as 4400 from the outside to the inside of the fuel region. Such a depression would indicate that most of the thermal neutrons produced in the reflector are absorbed in the first few centimeters of the fuel, and very few reach the inner regions. Thus,

Fig. 2 k_{eff} as a Function of Fuel Mass - Initial Design

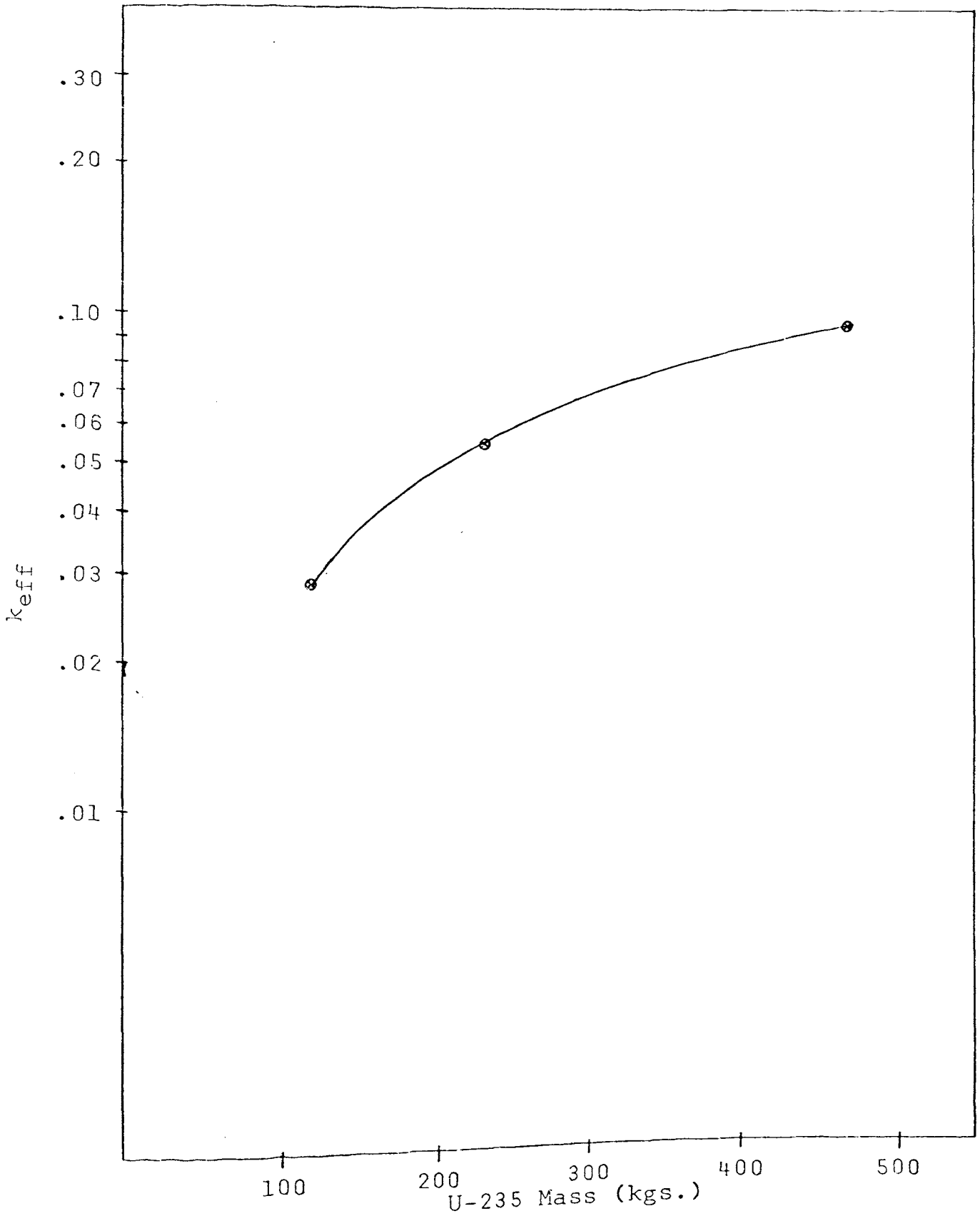


Fig. 3 Radial Thermal Neutron Flux - Initial Design

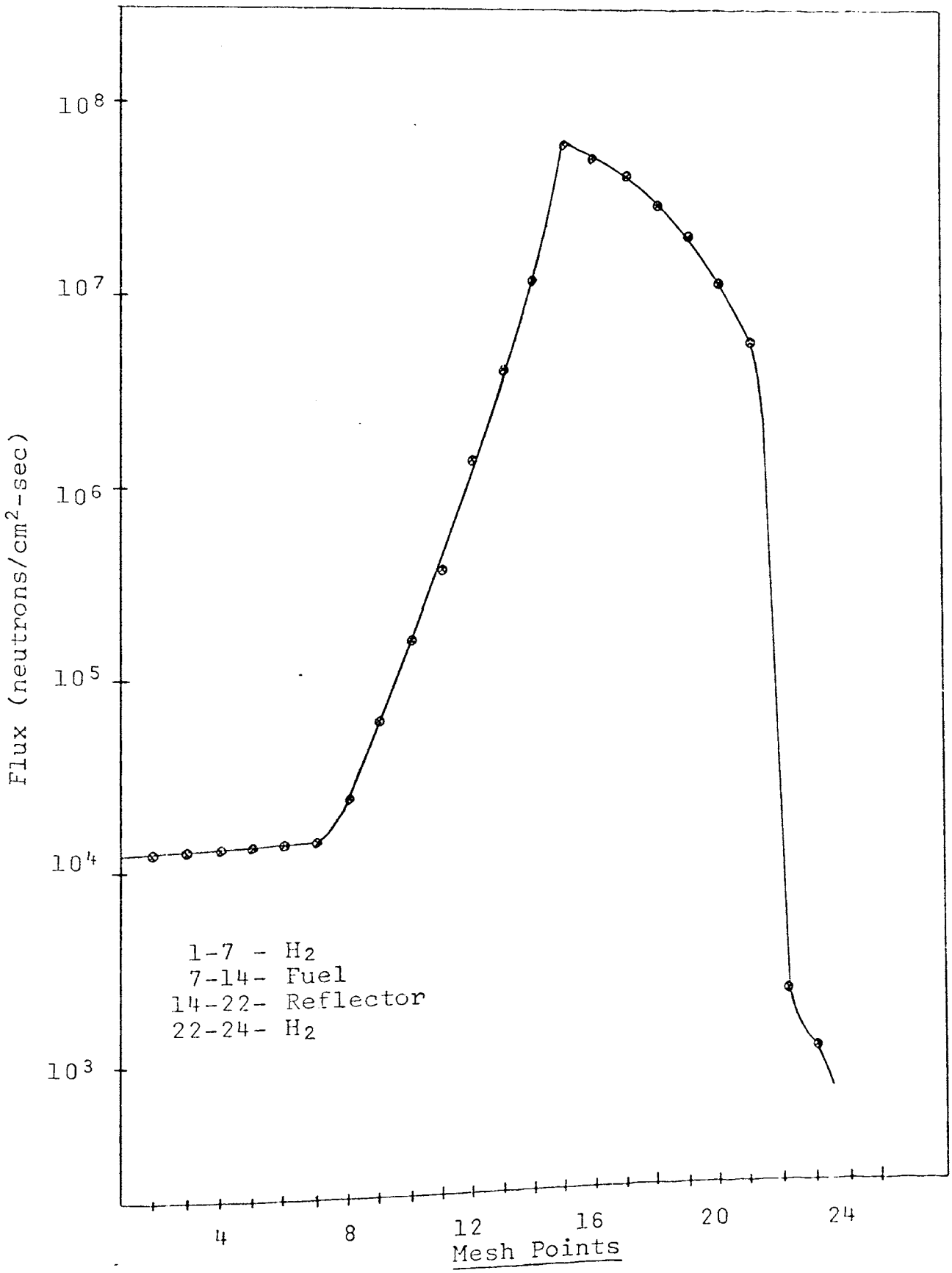
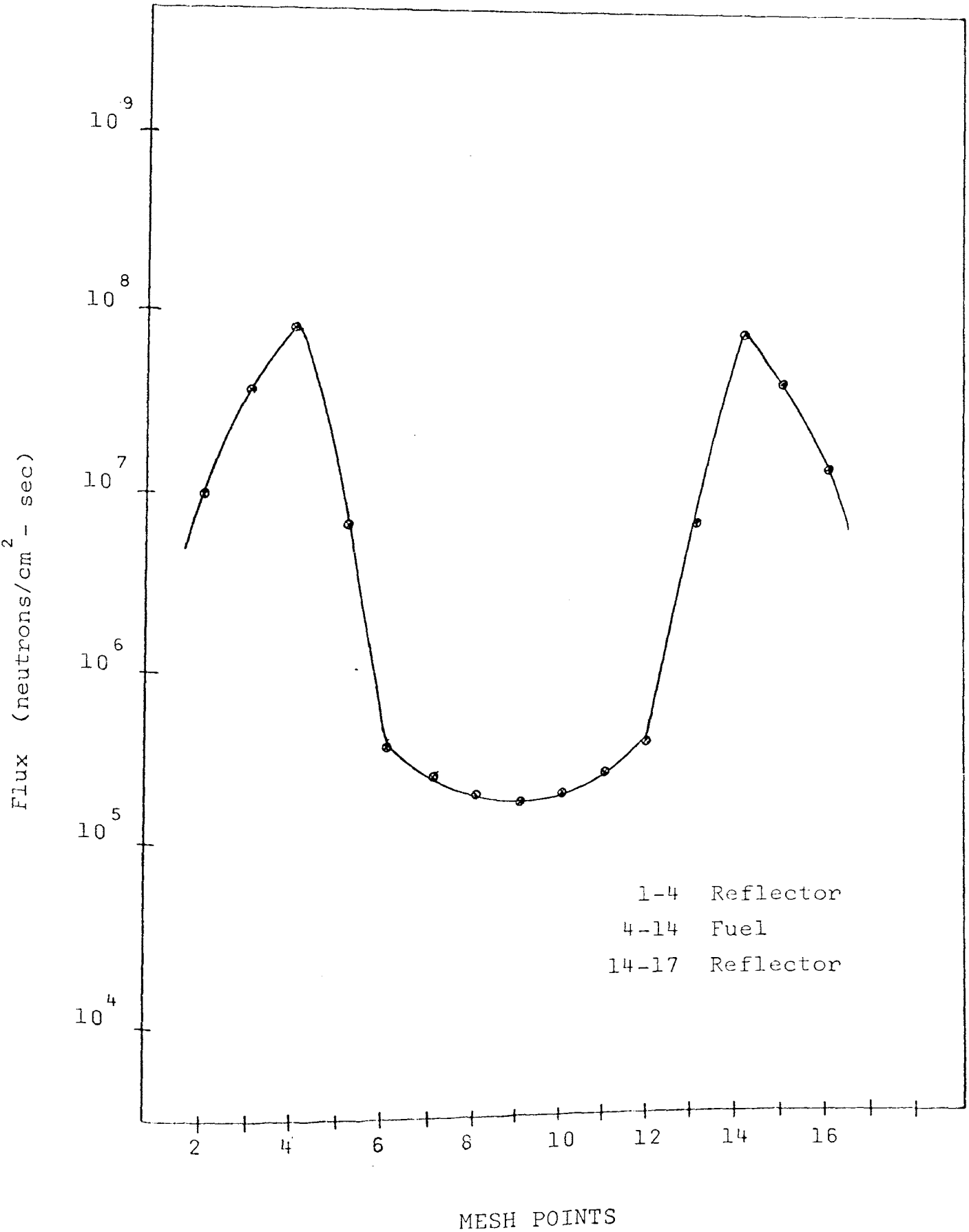


Fig. 4 Axial Thermal Neutron Flux - Initial Design



only a small portion of the fuel is being used effectively and as a result, the multiplication factor will not respond significantly to increases in the amount of fuel.

In order to make better use of the fuel, a beryllium oxide reflector was placed between the fuel and hydrogen collection chamber. It was theorized that the presence of this reflector would cause thermal flux peaks at both sides of the fuel region with a small thermal flux depression at the fuel center. A calculation was made with a six inch inner reflector and with the hydrogen collection chamber radius of fourteen inches. A large increase in the effective multiplication factor was found. The use of a six inch inner reflector caused k_{eff} to increase to .9765 with a fuel loading at 233 kgs U-235 and to .9906 with 467 kgs. The fluxes corresponding to this design are plotted in Figure 5. Figure 5 shows that a serious problem has developed. While the thermal flux at the inner and outer edges of the fuel are high, there is a severe flux depression in the center of the fuel. A flux depression (ϕ max/ϕ min) of 8421 occurs in one quarter inch of fuel. A flux depression of this magnitude is intolerable. In almost any power reactor, it is desirable to have a flat flux or at least a flux which is a slowly changing function of position. This promotes even fuel burnup and gives the best heat transfer. In the reactor proposed in this study, even fuel burnup is not a significant factor due to the short operating times and the fact that the vortex motion of the fuel will

Fig. 5 Radial Thermal Neutron Flux - with Inner Reflector

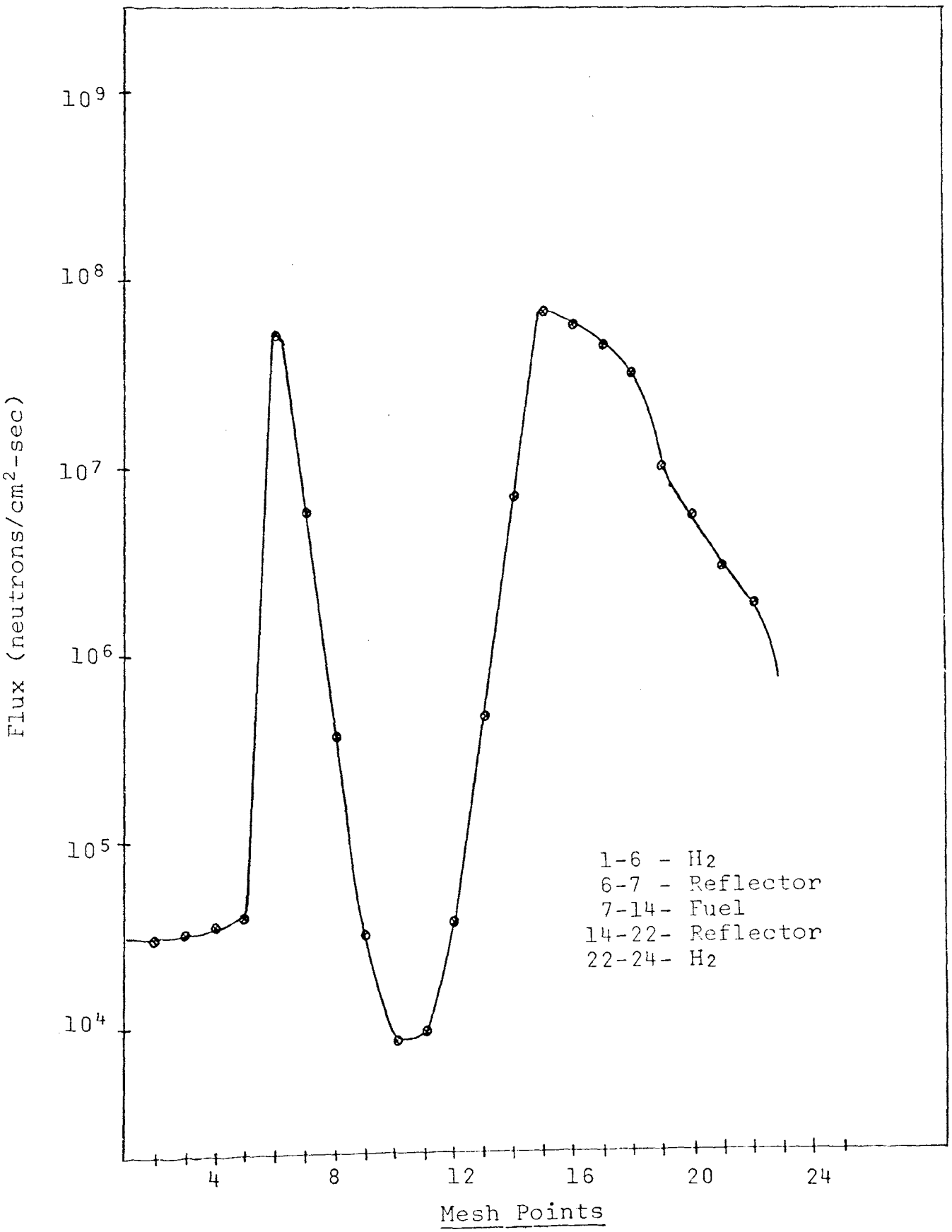
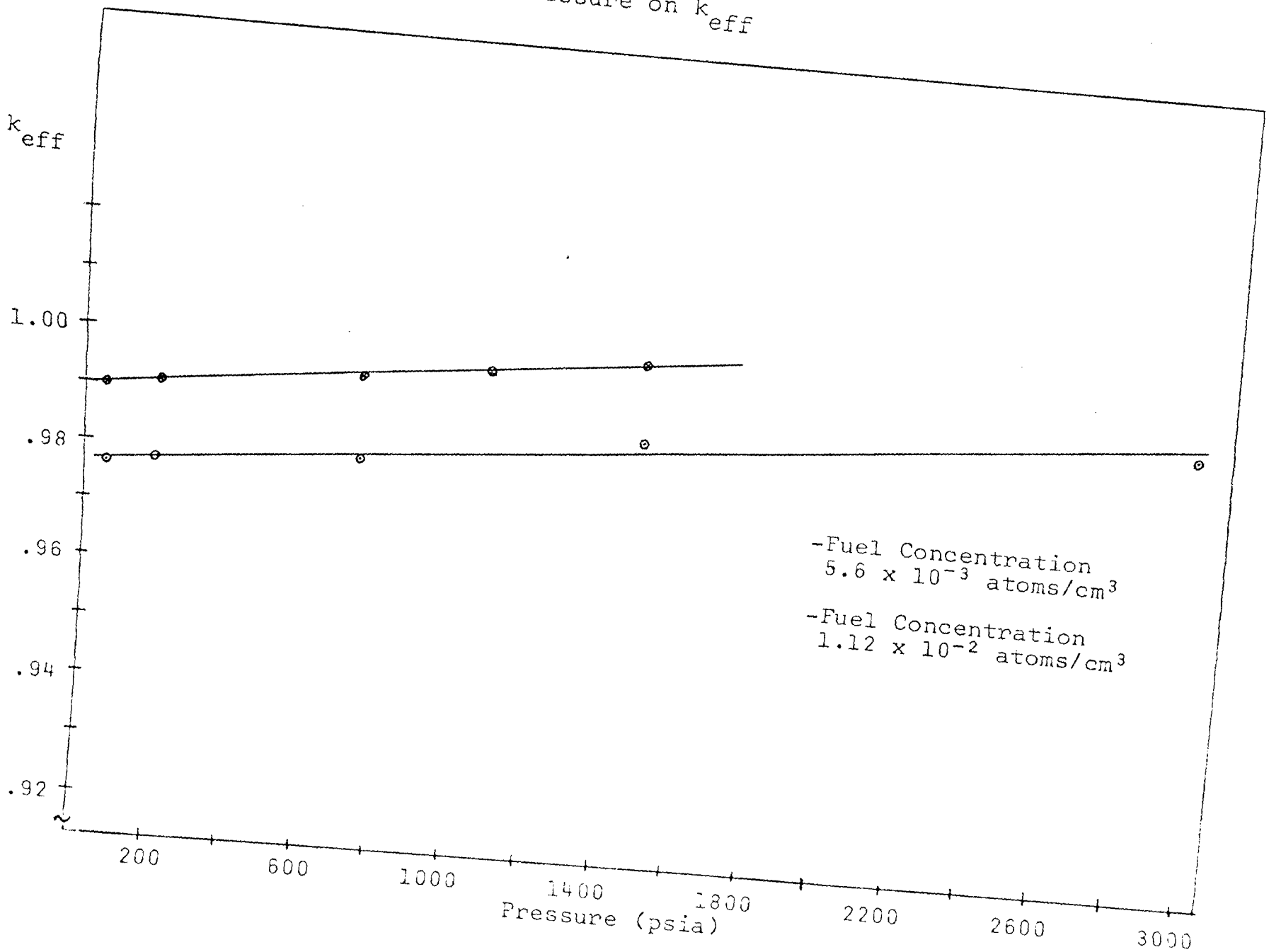


Table III. Hydrogen Density at Various Pressures

Hydrogen Density	Hydrogen Pressure
.0024	45 psia
.0096	183 psia
.0387	735 psia
.0591	1102 psia
.0795	1470 psia
.1590	2940 psia

All quantities are evaluated at 3500°R.

Fig. 6 Effect of Hydrogen Pressure on k_{eff}



counteract any uneven buildup of fission products. Maximum heat transfer is, however, most important in this system. The hydrogen coolant will only be in contact with the fuel for a short time. If the maximum fluxes occur only in a portion of the fuel, the amount of heat transferred is reduced.

In order to decrease the flux depression, more neutron moderation in the core was needed. Since the fuel system is in vortex motion, any substance added to the mixture will be forced from the fuel due to the centripetal force developed. Either another fuel mixture must be used or the density of the hydrogen must be increased.

It was thoughtthat increasing the hydrogen density by pressurizing the system would be the easiest way to increase moderation in the fuel. Trials were made with hydrogen pressures varying from 45 psia to 3000 psia. The various pressures and the corresponding hydrogen density and atom concentrations are listed in Table III. It was found for a fuel loading of 233 kgs, that hydrogen pressures above 3000 psia would raise the effective multiplication factor above unity thus making the reactor critical. Figure 6 shows the effects of various pressures on k_{eff}.

Increasing the hydrogen density did not eliminate the flux depression in the fuel. However, it did tend to reduce it. At a hydrogen pressue of 1470 psia, the ratio of the lowest and highest fluxes in the fuel is 90, still too high. The effects of coolant pressure on the flux depression are

shown in Figure 7.

Extrapolation of the curves in Figure 7 shows that with a fuel loading of 233 kgs U-235, a pressure of 7500 psia would be needed to reduce the flux depression (ϕ_{max} / ϕ_{min}) at such a high pressure is unfeasible and another method of decreasing the flux depression must be found.

An examination of Figure 5 offers a possibility. Since the flux is so low in the fuel center, the thickness of the fuel could be decreased, thus moving the two high flux points closer together and reducing the depression. Trials were made with 3/8, 1/4 and 1/8 inch thick fuel regions. The atom concentrations in the fuel were:

$$N_{25} = 5.6 \times 10^{-3} \text{ atoms/cm}^3$$

$$N_{Bi} = 1.05 \times 10^{-2} \text{ atoms/cm}^3$$

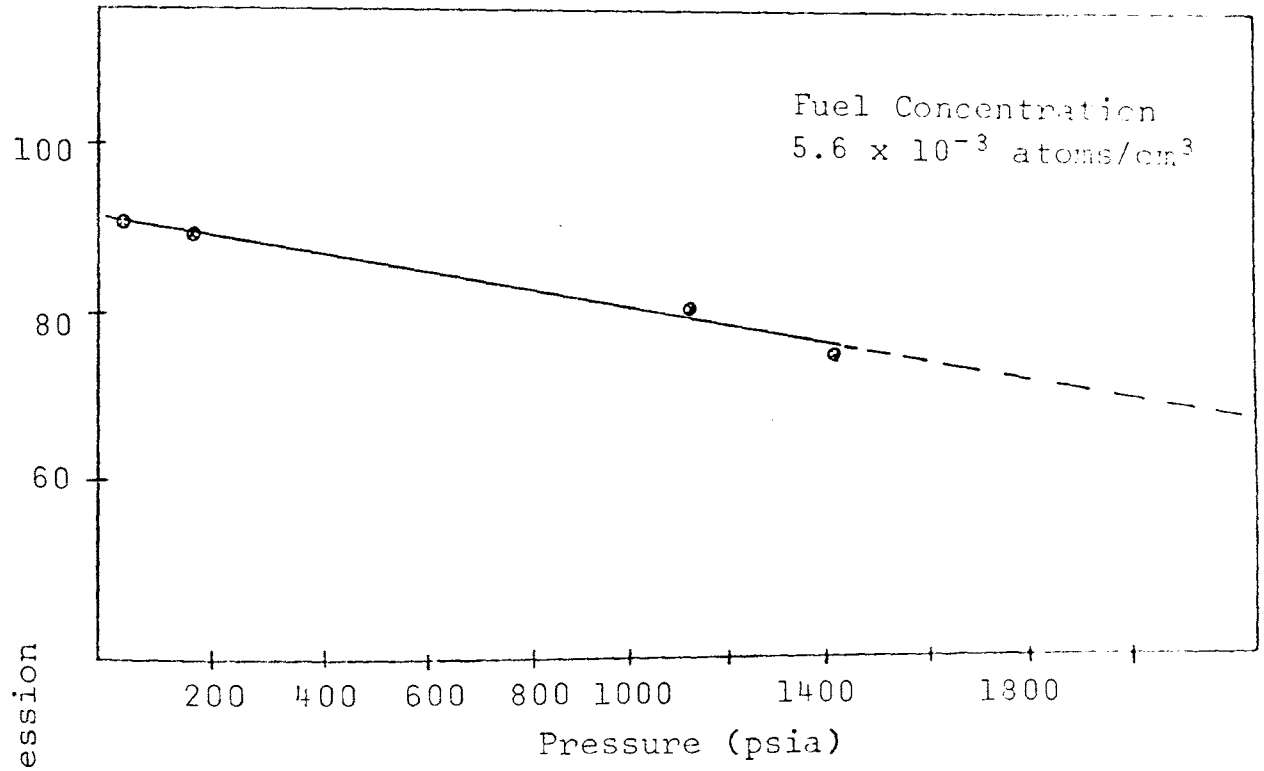
$$N_{H} = 8.12 \times 10^{-4} \text{ atoms/cm}^3$$

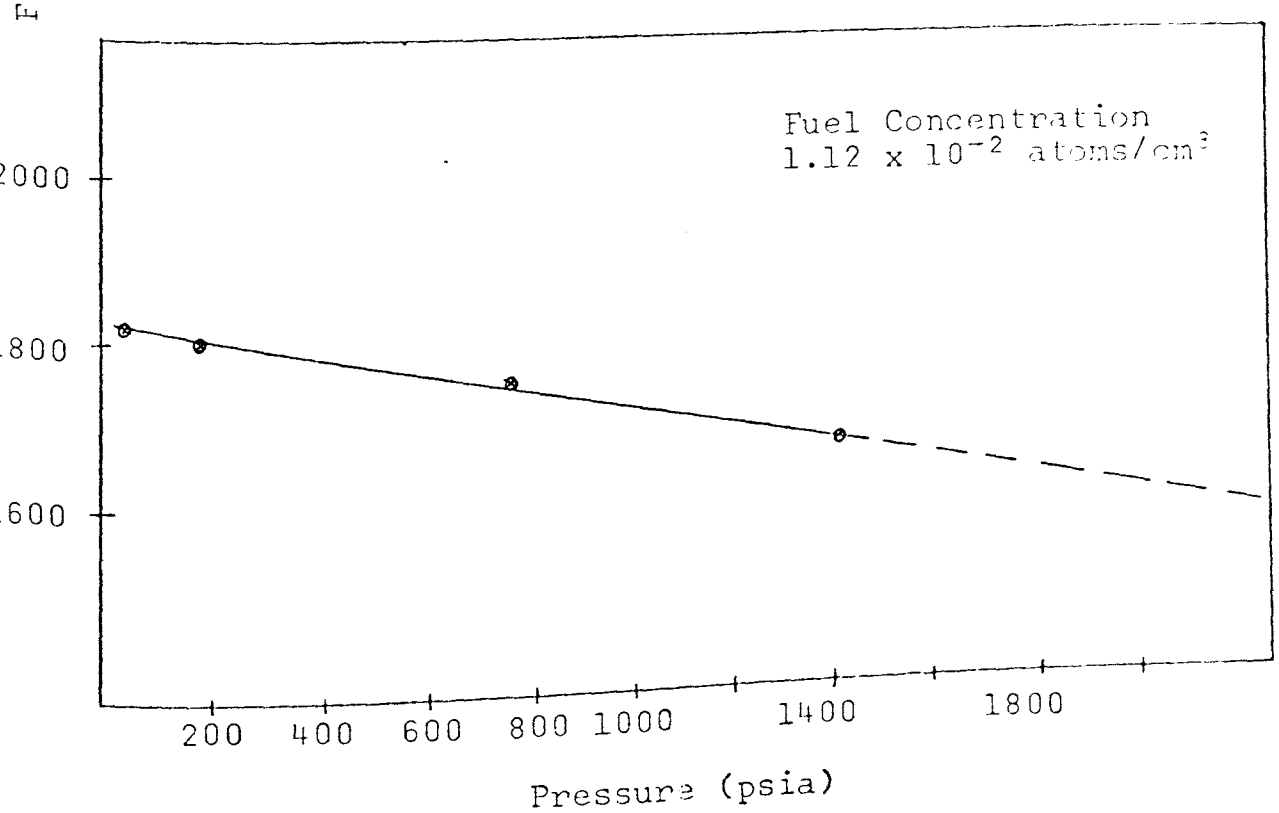
As the center portion of the fuel only makes a small contribution to the neutron flux, its removal should not seriously reduce the multiplication factor.

The results of this set of calculations verify the previous hypothesis. Figure 8 shows k_{eff} as a function of the fuel thickness. It can be seen that the multiplication factor increases as the fuel thickness is decreased from 1/2 to 1/4 inch and then decreases for thicknesses less than 1/4 inch.

Decreasing the fu thickness has also decreased the

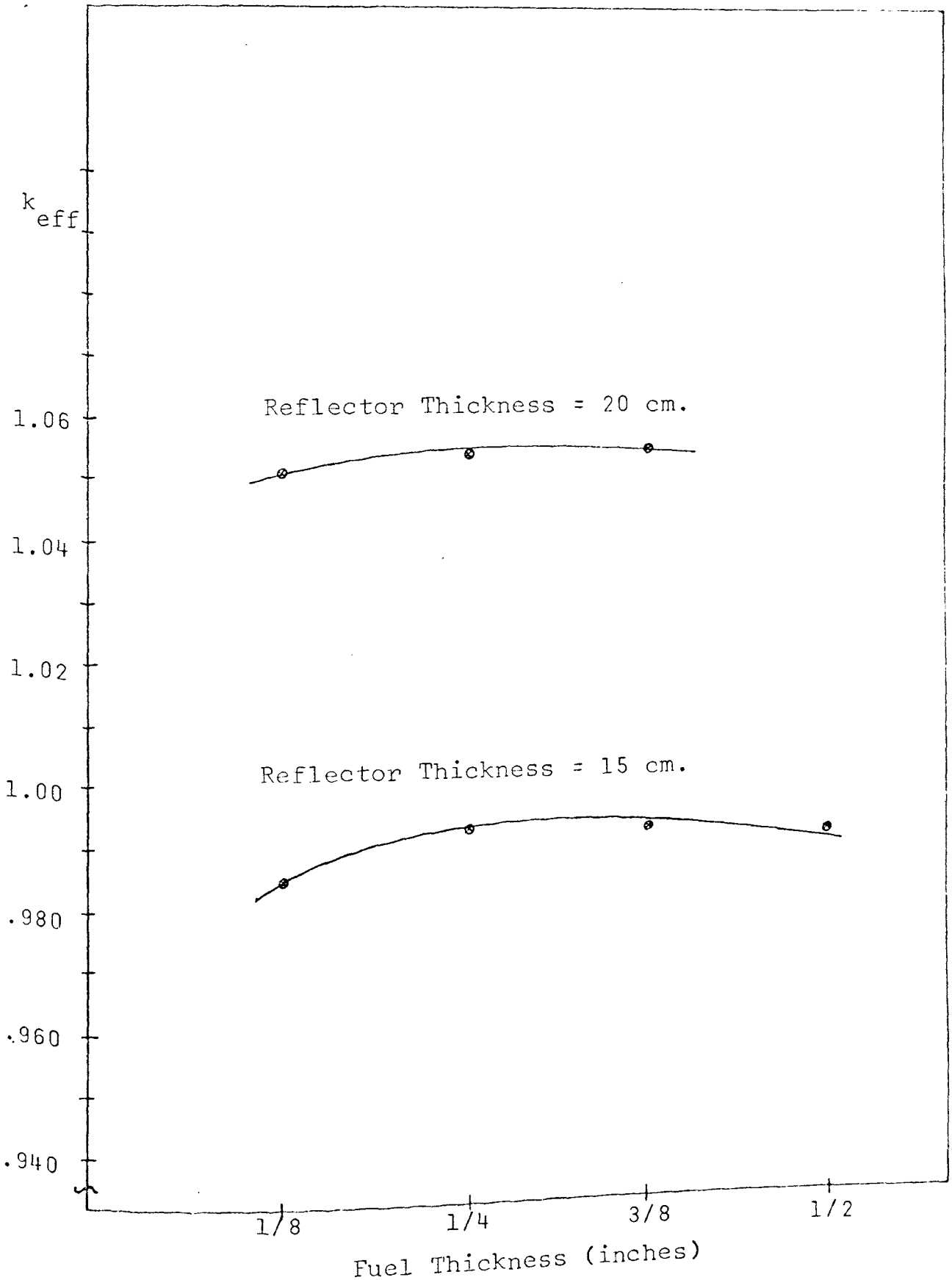
Fig. 7 Effect of Hydrogen Pressure on the Flux Depression





Flux Depression

Fig. 8 Effect of Fuel Thickness on k_{eff}



flux depression to an acceptable level. From Figure 9 it can be observed that the value of ϕ_{max} / ϕ_{min} is below ten for fuel thickness of 7/32 of an inch or less, and for a 1/8 inch fuel the ratio drops to less than four. Although the flux shape is still not flat, the curvature obtained with a 1/8 inch fuel is acceptable.

The multiplication factor for the 1/8 inch fuel was .9846. In order to increase this, five centimeters of beryllium oxide was added to the inner reflector. This raised k_{eff} to 1.051 while reducing the value of the flux depression to 3.85.

The hydrogen pressure in this system is 2940 psia. An attempt was made to reduce this by raising the U-235 concentration to 1.12×10^{2} atoms/cm^3 (25%) and reducing the hydrogen concentration to 4.06×10^{4} atoms/cm^3 (at 1470 psia). These concentrations resulted in a multiplication of 1.0630 for a 1/8 inch fuel and 1.0572 for a 1/16 inch fuel. The more concentrated fuel creates a flux depression of 10.8 at 1/8 inch and 4.0 at 1/16 inch.

Thus, two choices are available for the fuel size; a 1/8 inch region containing 12.5% U-235 with a hydrogen pressure of 2940 psia or a 1/16 inch region containing 25% U-235 with a hydrogen pressure of 1470 psia. The 1/16 inch region was selected mainly because of the lower hydrogen pressure. Figures 10, 11, 12 show the axial and radial fluxes in the reactor.

The final area to be studied is the amount of radial

Fig. 9 Effects of Fuel Thickness on the Flux Depression

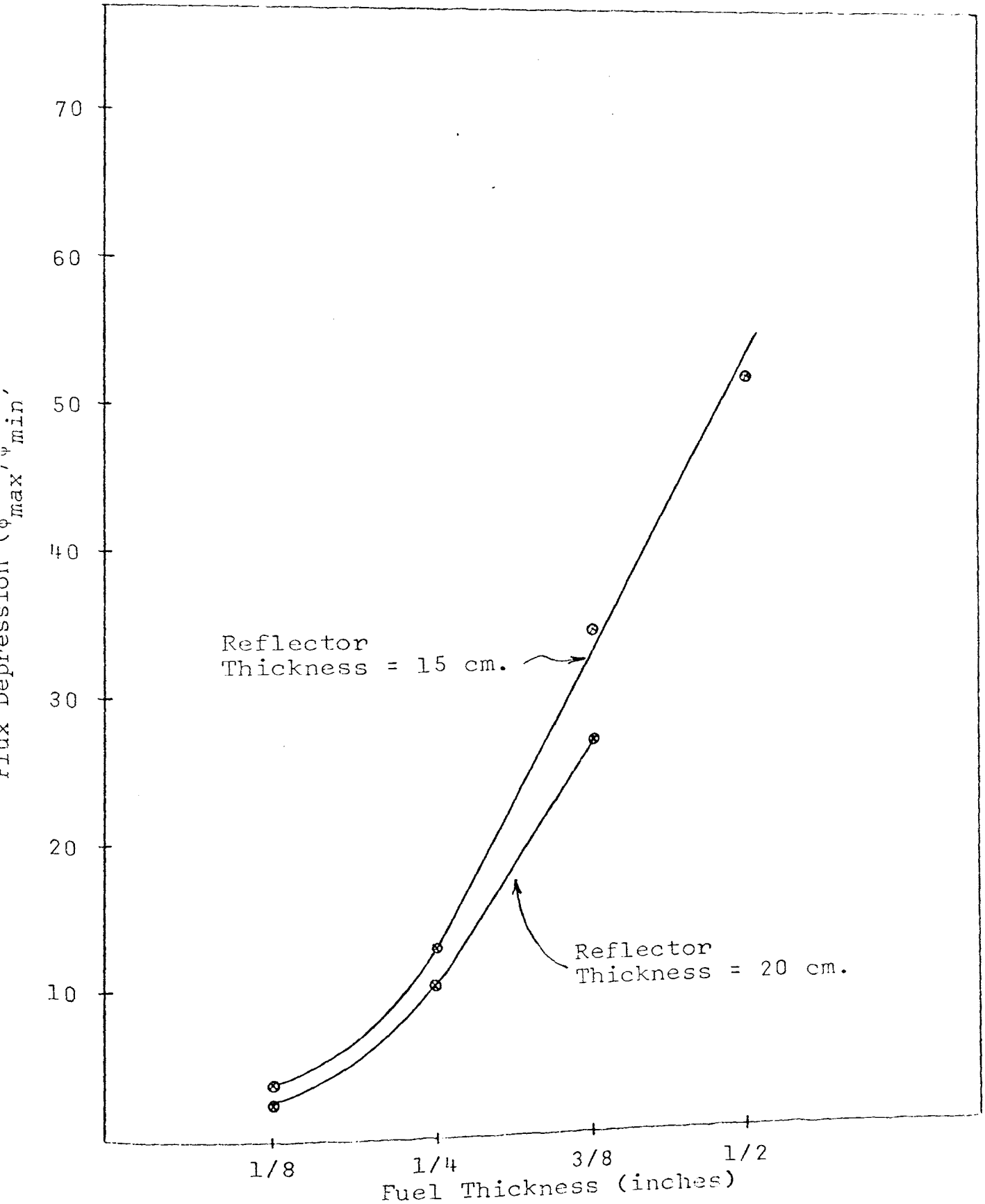


Fig. 10 Radial Thermal Neutron Flux - Final Design

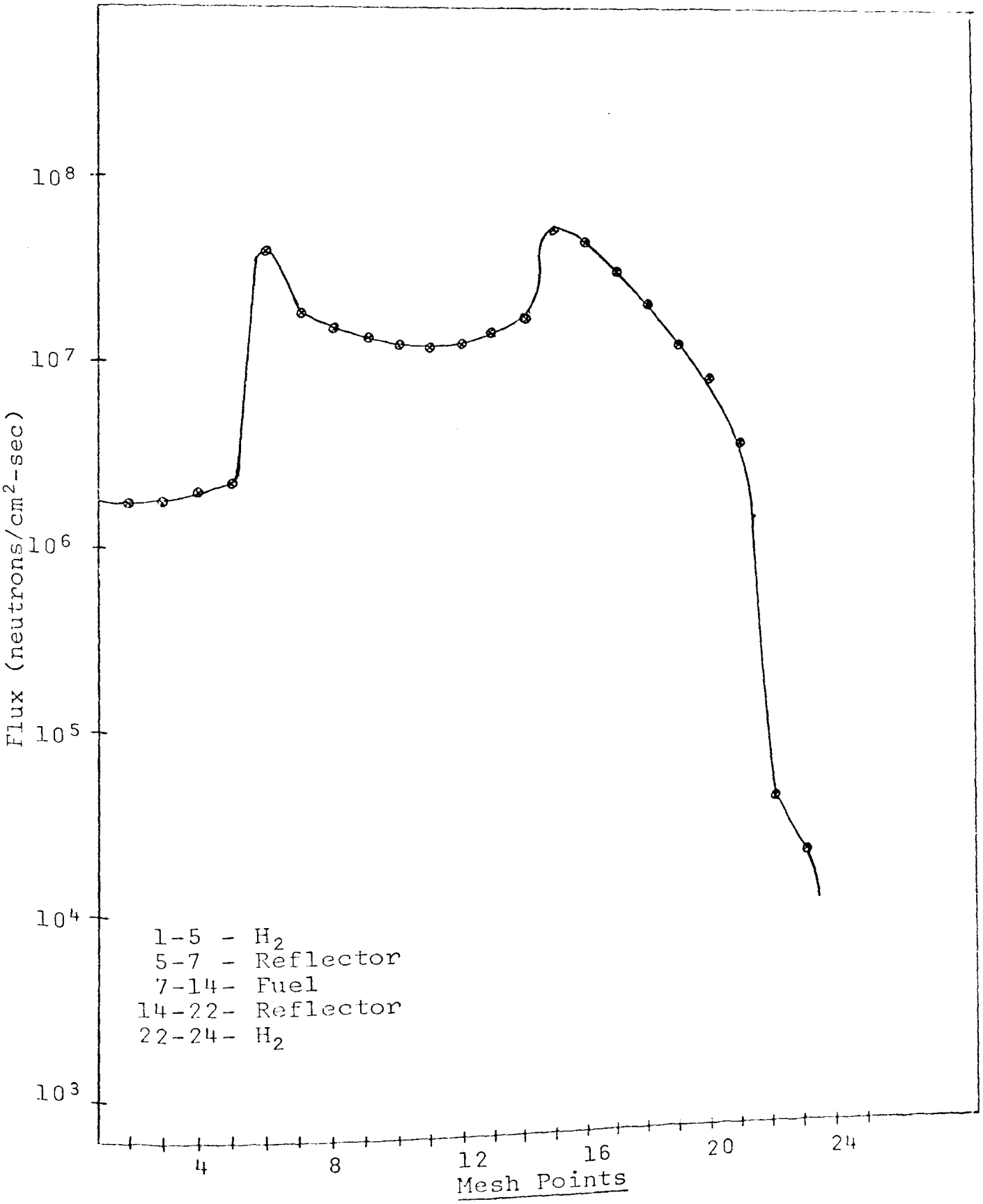
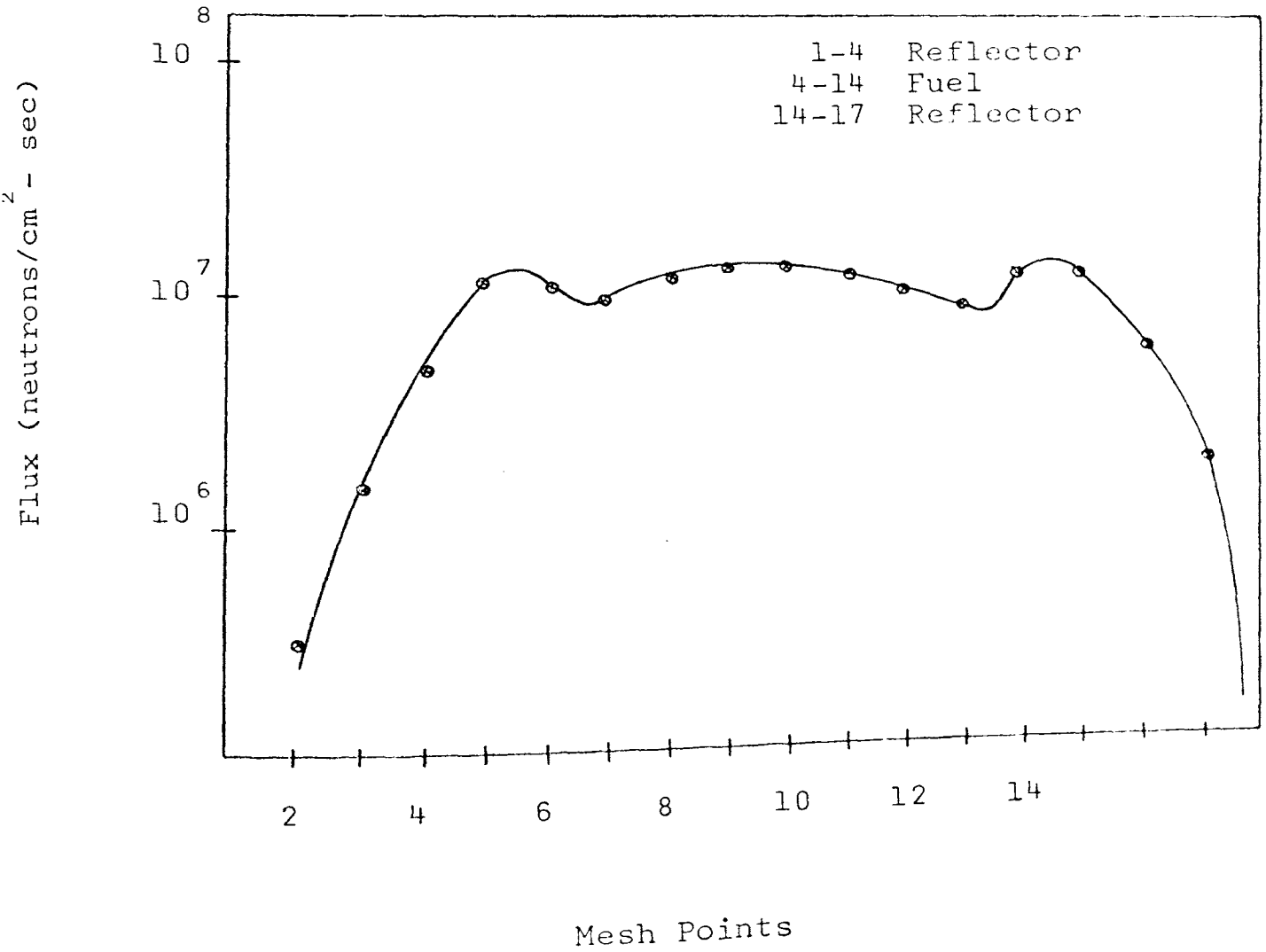


Fig. 11 Axial Thermal Neutron Flux - Final Design



reflector necessary to achieve the desired multiplication factor. In previous trials, a reflector thickness of 56 cm. was assumed. The beryllium oxide concentration was 6.67×10^{-2} atoms/cm^3. An examination of Figure 10 shows that the thermal flux peak in the inner reflector (thickness 20 cm.) is almost equal to that in the outer reflector. This would seem to indicate that size of the other reflector may be decreased without seriously affecting the neutron flux or k_{eff}.

Calculations were made on a 1/8 inch fuel region using reflector thicknesses of 56, 42, 28 and 14 centimeters. It was found that while decreasing the amount of reflector slowly decreased k_{eff}, the fluxes in the core decreased rapidly. This information is presented in Figures 14, 15.

Fig. 12 Radial Intermediate Neutron Flux - Final Design

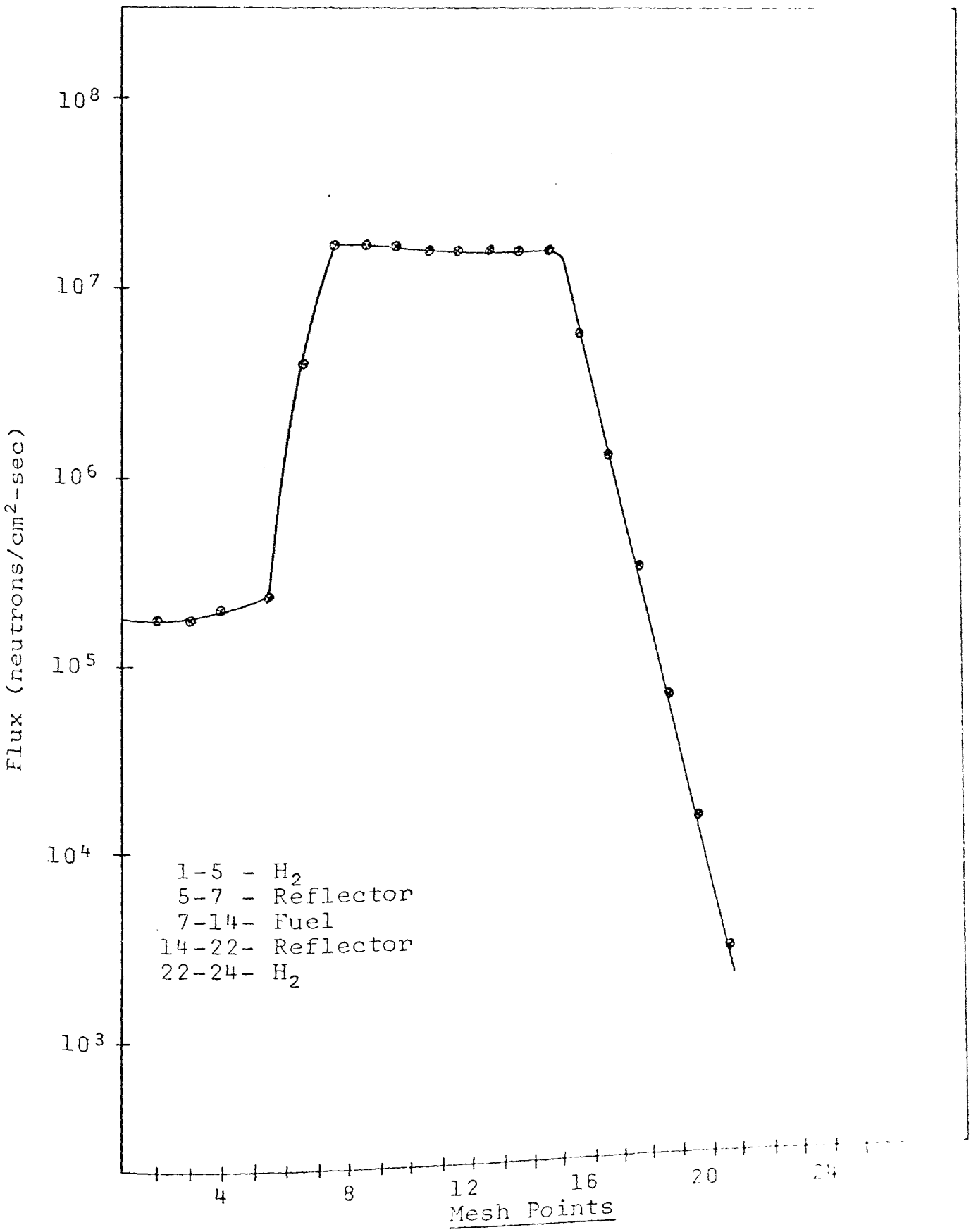
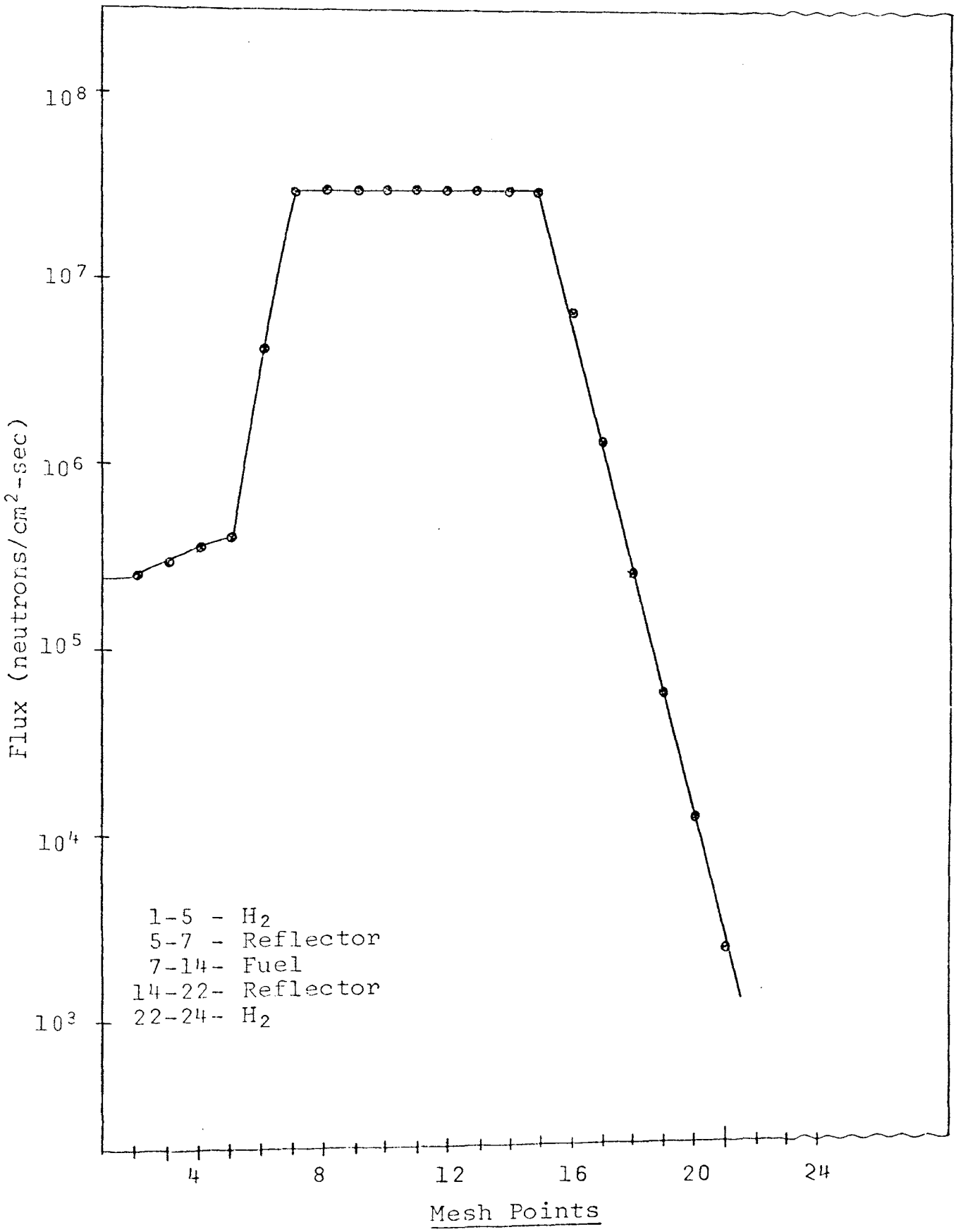


Fig. 13. Radial Fast Neutron Flux - Final Design



reflector necessary to achieve the desired multiplication factor. In previous trials, a reflector thickness of 56 cm. was assumed. The beryllium oxide concentration was 6.67×10^{-2} atoms/cm^3. An examination of Figure 10 shows that the thermal flux peak in the inner reflector (thickness 20 cm.) is almost equal to that in the outer reflector. This would seem to indicate that size of the other reflector may be decreased without seriously affecting the neutron flux or k_{eff}.

Calculations were made on a 1/8 inch fuel region using reflector thicknesses of 56, 42, 28 and 14 centimeters. It was found that while decreasing the amount of reflector slowly decreased k_{eff}, the fluxes in the core decreased rapidly. This information is presented in Figures 14, 15.

Fig. 14 Effect of Outer Reflector Thickness on k_{eff}

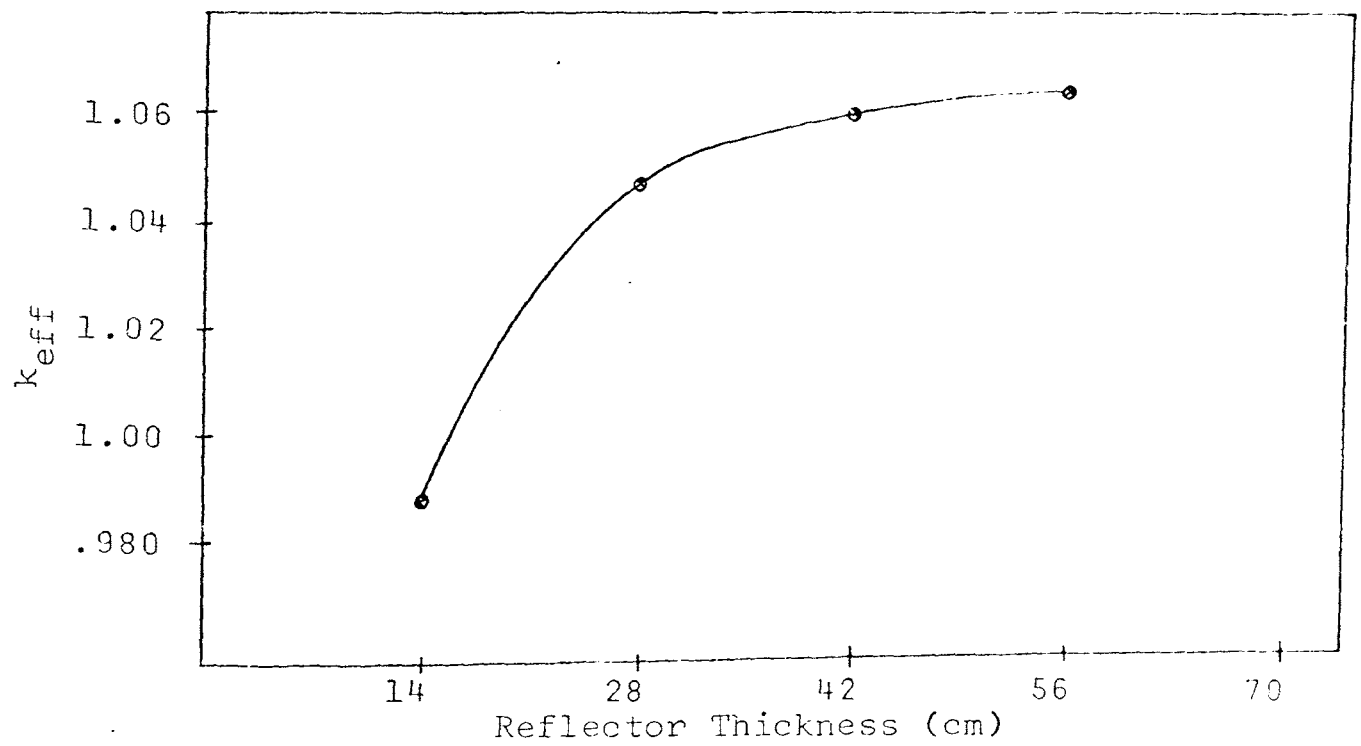
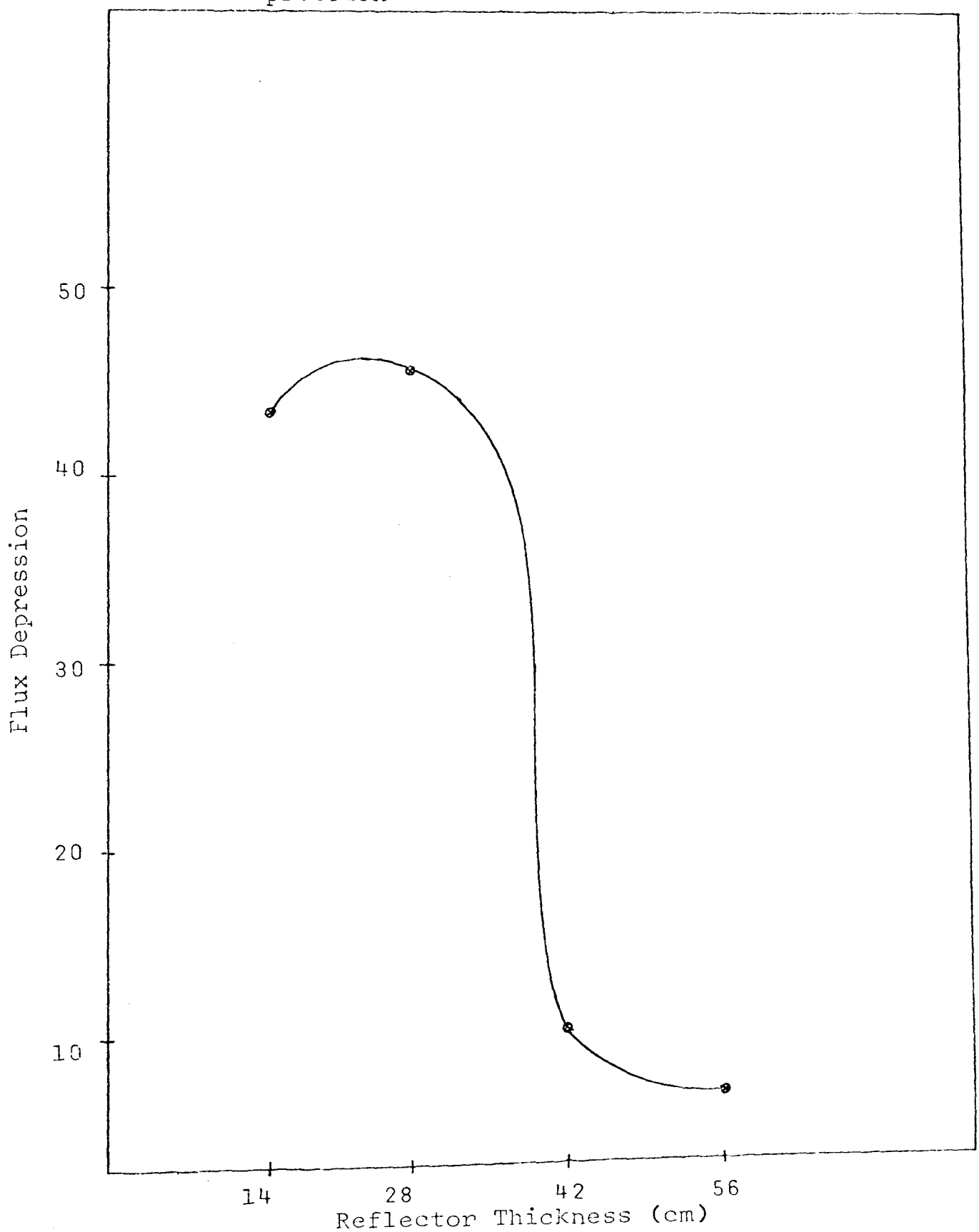


Fig. 15 Effect of Reflector Thickness on the Flux Depression



VIII. TWO PHASE VORTEX FLOW

Two phase vortex flow was investigated by J. F. Lafferty in 1966-1967. His work served as the guideline from which the various properties of the two phase vortex described in this work were found.

Although the basic principles employed in Lafferty's work were the same as those used in the reactor design, the actual systems varied greatly. His apparatus consisted of a five inch diameter vortex chamber surrounded by a one inch plenum ring. The height of the vortex chamber was also one inch. The gas-liquid mixture employed was air and water as compared to a hydrogen and liquid metal mixture used in the reactor. Lafferty's work was largely experimental although some theory was included.

In order to obtain a successful reactor design, five areas of vortex flow were investigated. These were the gaseous void fraction, pressure drop across the vortex, radial pressure of the liquid phase, radical velocity of the gaseous phase, and the conditions necessary to create a two phase vortex.

The two phase vortex is created by injecting the gaseous phase tangentially to the liquid region. This causes the liquid to rotate and the centrifugal action forces the less dense gas to the center where it is exhausted.

In order to determine the gas inlet velocity

necessary to rotate the liquid phase, the system was analyzed from an energy standpoint. In order for energy to be conserved, the kinetic energy of the rotating liquid must equal the kinetic energy of the gas at the inlet point minus any flow losses.

The kinetic energy of the rotating fuel is equal to the product of the moment of inertia and the square of the angular velocity. The moment of inertia (I) is found from equation 8.1 to be 4.95 slugs ft^2.

$$I = mR^2 ,$$

where: m = mass of body 8.1

R = radius

and the energy is $4.95\omega^2$. The kinetic energy of the gas is taken as $1/2mv^2$. Equating the two, the following is obtained: $4.95\omega^2 = 1/2mv^2$

There are three quantities which we must select, ω, v and m. It will be assumed that an angular velocity of two revolutions per second will be sufficient, in free space, to form the vortex. Various combinations of v and m were tried and a velocity of one hundred feet per second was selected. The resulting mass was found to be .063 pounds. The mass should be as low as possible to allow for longer operating times, however, the velocity should be reasonably low.

Once the rotational speed of the fluid has been determined, the pressure of the fluid on the container can be found. The centripital force is equal to the fluid mass

times the radial acceleration, a_r. The radial acceleration is found from $a_r = \frac{(2\pi R\omega)^2}{R}$ where R is the radius of the container. The radial acceleration was found to be 3479 in/sec^2 and the force on the container wall was computed as 2370 lb. From this information, the pressure was found to be .355 pounds per square inch at any point on the container wall.

Lafferty gives the following relation between pressure and the radius of the vortex.

$$\frac{P-P_e}{P_i-P_e} = (.00345)\ e^{5.76(r/r_o)}, \qquad 8.2$$

where:

P = Pressure in lb/in^2

e = Exit conditions

i = Inlet conditions

r = Radius at which P is to be found

r_o = Radius of the container

Taking the derivative with respect to r of equation 8.2 yields the differential pressure loss.

$$\frac{dp}{dr} = (P_i-P_e)\ \left(\frac{5.76}{r_o}\right)\ (.00345)\ e^{5.76(r/r_o)} \qquad 8.3$$

Assuming that P_i, the inlet pressure, is known, it is obvious that equation 8.2 cannot be solved without knowledge of either $\frac{dp}{dr}$ or the exit pressure.

In this case, however, an accurate determination of $\frac{dp}{dr}$ is not essential. The hydrogen pressure in the system is 1470 psia. This will be controlled by using orifices on the exit holes in the inner reflector. Thus, regardless of the pressure drop, the entire vortex system is kept at 1470 psia.

In his report, Lafferty reported that the void fraction is a function of the radius and the height of the vortex and is independent of the gas inlet pressure and the pressure drop across the vortex. As no mention was made as to whether or not the void fraction is a function of density, velocity, etc., it will be assumed that it is not. Lafferty determined experimentally the void fraction at various radii and has plotted the radial distribution of the average void fraction. From this information, an overall void fraction of .6 was assumed. It should be noted that the void fraction approaches unity at an r/r_o of .4. Thus, about one third of the vortex region is pure hydrogen. The dimensions of the fuel region were assumed to be ½ inch thick by forty eight inches high. The ½ inch is from the point at which r/r_o equals .4 and is assumed to be in the collection chamber region (region I).

Since the calculations in this section are based on fairly approximate theory, they may be in error. Due to the scarcity of information on two phase vortex flow, there seems to be no way of proving that the calculations are correct. An experimental apparatus would be one way to check their

validity. Such a device could be easily constructed and molten lead could possibly be substituted for the more expensive bismuth-uranium mixture.

IX. HEAT TRANSFER

One of the most important factors in determining the usefulness of a reactor for rocket propulsion is the amount of heat transferred to the propellant. It has been shown in equation 2.2 that the specific impulse of a rocket is largely a function of the propellant temperature. A reactor with poor heat transfer properties would be useless.

The majority of heat will be transferred from the fuel to the coolant by convection. The motion of the vortex will cause the hydrogen to bubble through the fuel. In order to determine the temperature rise on the gas, an individual bubble was studied. It will be assumed that the bubble is spherical in shape.

The general heat conduction equation for a sphere is (21)

$$\frac{\partial T}{\partial \Theta} = \alpha \left[\left(\frac{\partial^2 T}{\partial r^2} \right) + \frac{2}{r} \left(\frac{\partial T}{\partial r} \right) \right] \qquad 9.1$$

Where:

T = temperature

θ = time

r = radius

α = $k/\rho c_p$

k = thermal conductivity

ρ = material density

c_p = specific heat at constant pressure

This cannot be solved easily, however, a series of graphical solutions are presented in The Principles of

Heat Transfer by Kreith (21). These show both the temperature ratio, $(T_{r/r_o} - T_\infty) / (T_{\theta=o} - T_\infty)$ and the ratio of heat flow as a function of the Fourier modulus and the reciprocal of the Biot modulus.

$$\text{Fourier modulus} = \alpha\theta/r_o^2$$
$$\text{Biot modulus} = \bar{h}r_o/k \qquad 9.2$$

where:

$\bar{h}$ = the average unit surface conductance

The charts in reference 21 show that the temperature rise of the sphere is highest for large values of the Fourier and Biot moduli. Thus, for a maximum temperature rise, the ratios of θ/r_o^2 and $\bar{h}r/k$ should be close to or greater than unity. Taking the first case, the value of θ will probably be small. Even with a pressurized system, an individual bubble should not stay in the fuel for longer then one tenth of a second (2.78×10^{-4} hrs.). In order to obtain the desired value of the Fourier modulus, r_o^2 must be at least equal to 3×10^{-4} ft^2 (the value of α is 1.15 ft^2/hr). This corresponds to a bubble radius of .2 inches. Although no information could be found on the bubble size in a vortex system, it would seem that this size is much larger than that expected. Bubbles having a radius of less than .05 inches appear to be more reasonable. This is corroborated in a University of Florida report which includes a discussion of heat transfer in a vortex reactor, wherein the various parameters are optimized. These include flow rates, bubble sizes, pressures and velocities

(7). The author gives an optimum bubble size of 2.32×10^{-2} cm. or about .0091 inches. Bubbles of this size will result in a Fourier modulus greater than five hundred. A value of this magnitude will allow for an error in the choice of θ.

The large value of the Fourier modulus also eases the restriction on the Biot modulus. If it is assumed that the values of the conductivity and bubble radius are known, only $\bar{h}$ remains to be found. There are a number of empirical formulas for determing $\bar{h}$ under various conditions. In most of these the unit surface conductance is a function of Reynolds number, Prandtl's number and the thermal conductivity. In order to obtain a reasonable estimation of $\bar{h}$, the hydrogen was assumed to flow through tubes, whose surface area equals that of the bubble. Assuming a bubble diameter of one tenth of an inch, the tube diameter is .16 inches. The Reynolds number calculated using a gas velocity of fifty feet per second is 34900. All fluid properties were evaluated at two thousand degrees Farenheit.

Since the flow is turbulent ($N_{Re} > 4000$) the following equation was used to determine $\bar{h}$ (22).

$$\bar{h} = \frac{k}{D} \; (.027) \; (N_{RED})^{.8} \; (N_{Pr})^{1/3} \; \left(\frac{\mu}{\mu_s}\right)^{.14} \qquad 9.3$$

Where:

N_{Pr} = Prandtl number = .71

μ_s = Dynamic viscosity, evaluated at the pipe surface temperature - 1.74 lb/ft-sec

Solution of equation 8.3 results in a value of 1045 BTU/hr-ft^2-°F for $\bar{h}$. It should be remembered that this value is very approximate due to the assumption necessary. Since the possibility of error is great, the worst situation will be assumed and $\bar{h}$ will be reduced to 10.45 BTU/hr-ft^2-°F. Using this value, the Biot modulus is 1.74. When this is applied along with Fouriers modulus, to Krieth's chart, $\frac{T_{r/ro} - T_\infty}{T_o = o - T_\infty}$ is found to be close to zero, indicating that $T_{r/r_o} - T_\infty$ is small. Thus, the final temperature of the gas will be almost equal to the fuel temperature of four thousand degrees Farenheit.

X. CONCLUSIONS AND RECOMMENDATIONS

The reactor studied in this report has been shown to meet the basic requirements of reactor physics and heat transfer.

The final design calls for the use of a 1/16 inch fuel region containing 25% U-235, 25% bismuth and 50% hydrogen. The fuel will have an inner reflector of 20 cm. and an outer reflector of 56 cm. The axial reflector thickness is 45 cm. A reactor of this design will have a k_{eff} of 1.057 and the ratio of ϕ_{max}/ϕ_{min} of less than four.

The use of a pressurized system (1470 psia) will prohibit any dissociation by the hydrogen molecules. This is one undesirable feature of the reactor.

The hydrogen will leave the fuel at approximately 4000°F and when exhausted through the proper nozzle, produce a specific impulse of 835 seconds, more than twice that of the best chemical rocket.

A number of areas must be investigated before this design could be considered for operation. Some of these would include:

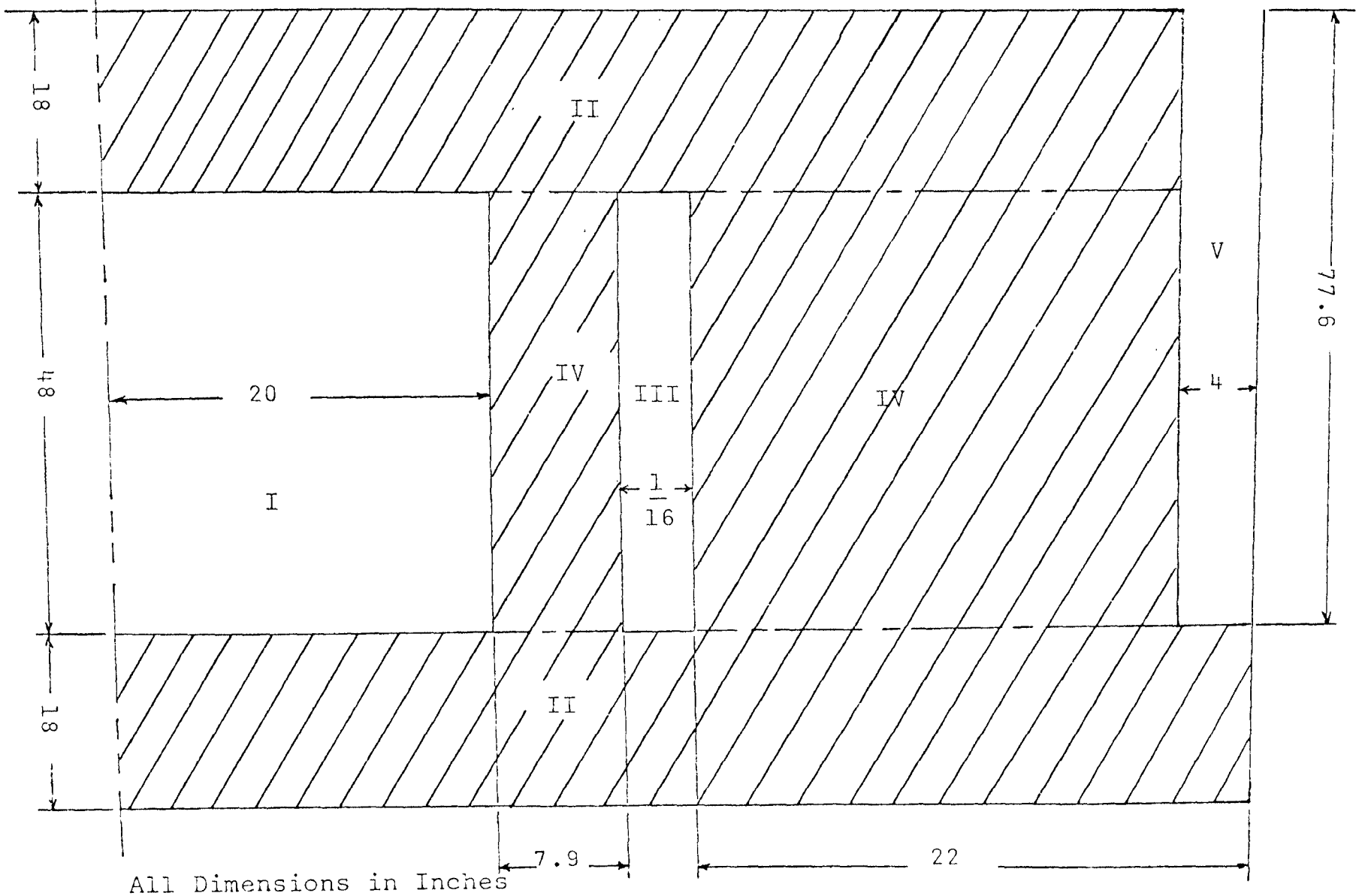
1) A study of the properties of beryllium oxide at high temperatures.

2) A study of two phase vortex flow with a heavy liquid metal and hydrogen.

3) The effects of motion on the vortex.

4) A detailed heat transfer study.

Fig. 16 Cross Sectional View of the Reactor - Final Design



All Dimensions in Inches

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VITA

Robert Philip Schmitt was born on July 20, 1944, in Plainfield, New Jersey. He received his elementary and secondary education in the New York City Public School System. In 1966 he was graduated from the State University of New York - Maritime College with a B.S. in Nuclear Engineering and was licensed by the United States Coast Guard as a Third Assistant Engineer.

He worked for the following six months as a Marine Engineer for various private shipping companies.

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He married the former Diane Dowd in July,1967.